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Direct Measurement of the Intensity Distribution in Fresnel Diffraction Patterns

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IN the course of an investigation of a photoelectric system for the measurement of small light intensities, the possibility occurred to us of measuring directly the intensity distribution in Fresnel diffraction patterns. The results obtained are in such good agreement with the predictions of the Fresnel theory that it seemed worth while to offer them for publication. Although a number of photographs of various diffraction patterns have appeared in the literature from time to time, very few investigators have actually measured the intensity distribution in the patterns. Lyman,[1] using a photographic method, has made a series of measurements of the intensity ratios of maxima to minima in a straight edge diffraction pattern. His measured intensity ratios agreed within 4 percent with the corresponding ratios taken from the theoretical pattern. This method involves very careful calibration of the photographic plates, and it would be quite difficult to apply in some cases where the intensity range involved is quite large. Use of a photo-cell and amplifying circuit provides a method subject to somewhat better control than the photographic method and a method which is directly applicable to a large intensity range.

METHOD OF MEASUREMENT

The energy distribution in the various diffraction patterns was measured by means of a photoelectric cell in conjunction with a sensitive amplifying circuit. The amplifying circuit used was the DuBridge-Brown[2] balanced circuit employing a General Electric FP54 electrometer tube. The diffraction camera consisted of a light-tight box approximately 570 cm long with diffracting slit fitted at one end. The other end of the camera was arranged such that it could be fitted with translucent screen for visual observation, a plateholder for photographing the pattern, or a metal box containing photo-cell, battery, electrometer tube, and grid resistor. This box, containing photo-cell and other equipment, was mounted upon a carriage which could be moved across the pattern by means of a lead screw of 1 mm pitch. In order to obtain reasonable resolution, the aperture of the photo-cell was limited to a slit 0.5 mm wide and 2 cm long. Some little difficulty was encountered in obtaining a monochromatic light source of sufficient intensity to give measurable patterns; but eventually, it was found that the General Electric mercury arc No. H-3 of the capillary type was satisfactory. Three glass filters were used with the arc: Corning G34Y and Cambridge Botanical supply Nos. 8 and 16, allowing transmission of the mercury green line at 5461A. The No. 16 filter was necessary to eliminate quite an appreciable band of continuous radiation in the red. In order

[1] T. N. Lyman, Proc. Nat. Acad. Sci. **16**, 71 (1930).

[2] Lee A. DuBridge and Hart Brown, Rev. Sci. Inst. **4**, 532 (1933).

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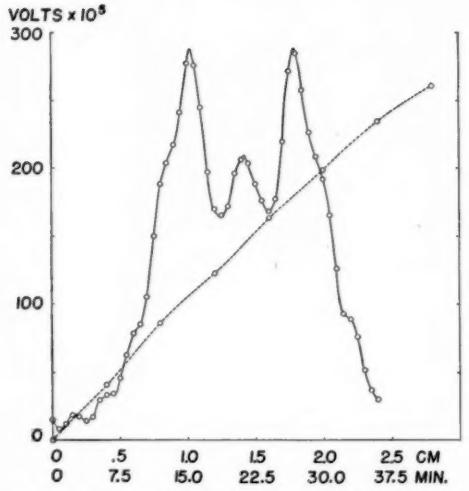

FIG. 1. Intensity distribution in a single-slit diffraction pattern for which $\Delta v = 4.34$. The broken line shows the drift in the receiving apparatus during the course of the measurements. The solid line shows the measured intensity distribution after correction has been made for the drift.

to approximate the line source required by the Fresnel theory, a narrow slit was illuminated from behind by the filtered arc. The source slit itself was about 50 cm from the diffracting slit. The diffracting slit was adjusted by using small, accurately-ground, roller bearings as separators. The bearings were then carefully measured to obtain the slit width.

The low intensity of the diffraction patterns when observed visually, and, in addition, the limited photo-cell aperture required to insure resolution, made it evident that only very feeble photo-cell currents would be obtained. Subsequent measurements showed that the photo-currents were between 10^{-13} and 10^{-15} ampere. However, currents of this magnitude are well within the range of the amplifying circuit employed. Using a grid input resistor of 10^{10} ohms and a Leeds and Northrup type H galvanometer, a current sensitivity of 5×10^{-16} amp./mm was obtained. A potentiometer was incorporated in the grid circuit, and the system used as a null instrument. The photo-current resulting from radiation incident on the photo-cell was then measured by measuring the potential drop produced across the grid resistor. The photo-cell employed was a Visitron type 53AV vacuum cell. This, with a C battery of 45 volts, was connected directly to the grid input resistor. The photo-cell was tested carefully and found to give a linear response over the entire range employed.

Some troubles were encountered in building the amplifying circuit, but this is apparently not at all unusual. The high sensitivity of the circuit requires that the resistors be carefully selected, the supply batteries in good condition, and the shielding of the circuit complete. Even after meeting the above requirements as well as possible with the equipment available, a small amount of drift remained in the galvanometer circuit. This drift, however, was sufficiently uniform and could be reduced to such an extent that the data was easily corrected. The drift could probably have been further reduced by evacuating the box containing photo-cell, electrometer tube and grid resistor.

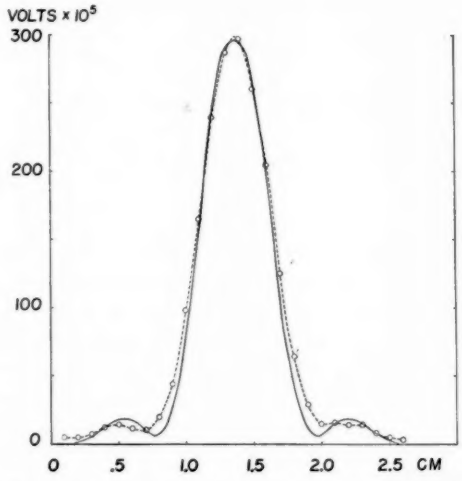

FIG. 2. Measured and calculated intensity distributions for a single-slit diffraction pattern, $\Delta v = 1.40$. The broken line shows the measured intensity distribution and is a mean of three separate sets of measurements. The solid line is the calculated intensity distribution adjusted to conform with the experimental conditions.

EXPERIMENTAL RESULTS

Three single-slit diffraction patterns have been measured in detail. The slit widths were chosen such that the patterns differ quite markedly in form, ranging from the familiar, narrow-slit pattern to the more complex pattern made up

of two principal maxima and two secondary maxima.

In measuring a diffraction pattern it was necessary to arrange the data in such a manner that correction could be made for the residual drift in the amplifying system. This was done by recording not only the potentiometer reading for a given photo-cell position but also the time at which the potentiometer setting was made relative to the first measurement of the set. In addition the photo-cell was arranged with a shutter, and every eighth reading was taken with the shutter closed. These dark readings were thus taken against time alone and were independent of the particular pattern being measured. After an entire pattern was measured the dark readings were plotted against time and the points ob-

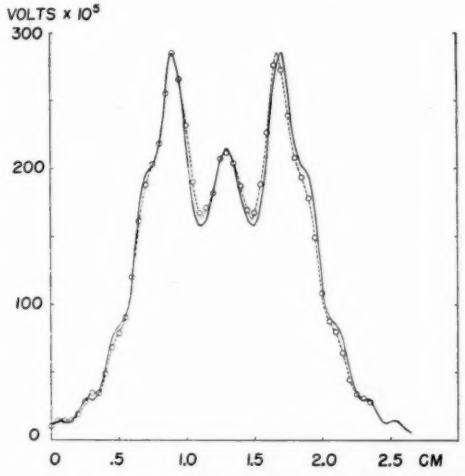

FIG. 3. Measured and calculated intensity distributions for a single-slit diffraction pattern, $\Delta v = 4.34$. The broken line shows the measured intensity distribution and is a mean of two separate sets of measurements. The solid line is the calculated intensity distribution adjusted to conform with the experimental conditions.

tained connected by a smooth curve. This curve was taken as a measure of the drift of the amplifying system. The uncorrected voltage readings for the diffraction pattern were then corrected by obtaining the drift correction from the drift curve corresponding to the time at which the measurement was made. To give an idea of the magnitude of the drift during a typical set of measurements, one of the drift curves is reproduced as the broken curve in Fig. 1. The drift was quite uniform as is apparent from this curve and was usually of about the same magnitude.

The solid curve in Fig. 1 shows the measured points for one pattern after the drift correction had been made. Voltage readings were taken every half-millimeter, the pattern having an extent of about 2.5 cm. The distance from the source slit to the diffracting slit was 53.5 cm and from diffracting slit to photo-cell aperture 567.9 cm. For this particular pattern the diffracting slit had a width of 1.586 ± 0.001 mm.

It was quite difficult to obtain a completely symmetrical pattern even after careful alignment of the apparatus. Patterns which appeared symmetrical when observed visually, invariably showed some small variations when actually measured. To correct, in part, for this asymmetry each pattern was measured several times and the average ordinates taken. Individual measurements of all intensities giving values in excess of 20×10^{-5} volt did not differ from the mean by more than 4 percent, and in a large number of cases all the measurements agreed within 2 percent. No measurements within a given set were discarded. If some readings were obviously in error the entire set was discarded.

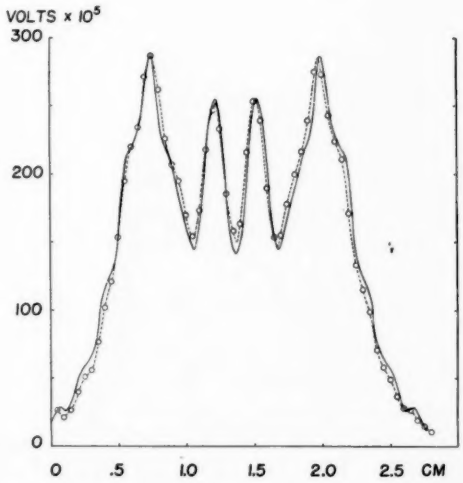

FIG. 4. Measured and calculated intensity distributions for a single-slit diffraction pattern, $\Delta v = 5.42$. The broken line shows the measured intensity distribution and is a mean of three separate sets of measurements. The solid line shows the calculated intensity distribution adjusted to conform with the experimental conditions.

Figure 2 shows the measured points with a very narrow diffracting slit, here 0.512 ± 0.001 mm. This pattern is very similar to a Fraunhofer single-slit pattern, with the exception that the intensities at the minima do not go to zero. The points appearing are the mean of three separate sets of measurements. Fig. 3 shows the measured points with a wider diffracting aperture, 1.586 ± 0.001 mm. The plotted points are the mean of two separate sets of measurements, one set being reproduced in Fig. 1. Fig. 4 shows the measured points with a diffracting slit of width 1.979 ± 0.001 mm. The plotted points are a mean of three separate sets of measurements.

Comparison with Theory

In order to compare the intensity distribution in the measured patterns with the distribution predicted by the Fresnel theory, it was necessary to construct theoretical patterns and superimpose them upon the measured patterns. However, since the experimental set-up only approximated the requirements of the theory, it was necessary to make some corrections before the actual comparison was made. The Fresnel theory assumes a line source of radiation, and gives an intensity distribution in the diffraction pattern which is continuous; this would require for measurement an instrument of unlimited resolving power. These requirements were only approximated in this work. For every pattern the source slit was adjusted until a pattern of reasonable sharpness was obtained and yet of sufficient intensity to allow measurement. This meant a source of finite width rather than a line source. Also, in every case the photo-cell aperture was 0.5 mm which resulted in an actual measurement of a section of the pattern 0.5 mm wide. It seemed simpler to make adjustments for these variations by correcting the theoretical curves and thereby obtaining curves truly representative of the experimental arrangement. By using the dimensions of the apparatus, theoretical patterns were constructed. These were obtained in a detailed fashion by the use of an extended table of Fresnel integrals compiled by C. M. Sparrow.[3] To adjust for the finite width of the source slit it was assumed that the measured pattern consisted of a group of superimposed individuals as given by a group of line sources covering the extent of the source slit. The predicted intensity distribution in the measured pattern was obtained by determining the spread of the patterns due to the source slit; and then, measuring the area of sections of this extent on the theoretical curves with a planimeter. This area as measured with the planimeter was taken as being proportional to the intensity at the midpoint of the section measured. In Figs. 2 and 3 the source slit was 0.07 mm wide, giving a spread of 0.7 mm in the measured pattern. In Fig. 4 the source slit was 0.04 mm. The result of this correction was a theoretical intensity distribution curve representative of the actual diffraction pattern measured by the photo-cell. However, before this curve could be compared with the measured ones, it had to be further corrected for the width of the photo-cell aperture. This correction was made in a manner similar to the source slit correction; i.e., the intensity of the light falling upon the photo-cell was taken as proportional to the area of a section of the curve having the width of the photo-cell aperture. As was to be expected, these corrections tended to raise the minima and lower the maxima and, in general, smooth out the theoretical curves. These corrections were somewhat tedious to make, but it did not seem that a legitimate comparison could be made otherwise.

The final corrected curves were now adapted to the same scale as the measured patterns. This was accomplished by comparing a corresponding point of the two patterns and obtaining a scale constant and an intensity ratio. This amounted to bringing the measured and theoretical curves into coincidence at one point. The point where the two curves were brought into coincidence was taken slightly to the right of the maximum in Fig. 2 and directly at a maximum in Figs. 3 and 4. These adjusted theoretical curves are the solid lines in Figs. 2, 3 and 4.

In general the agreement between measured and theoretical patterns is quite good. The poorest results are those obtained where the intensities are very low as there the experimental error in measurement was quite large. This is particularly true for the narrow slit pattern in

[3] C. M. Sparrow, *Table of Fresnel Integrals* (Edwards Brothers Inc., Ann Arbor, 1934).

Fig. 2, where the secondary maxima are evidently in error. Potentiometer settings could be made to 2×10^{-5} volt, which gave an accuracy of about 1 percent for the high values but an accuracy of only about 6 or 7 percent for the low values. However, by taking the average of several measured patterns this error should be reduced somewhat. In Figs. 3 and 4 the ratios of principle maxima to minima for the measured and theoretical patterns are somewhat in error. The disagreement amounts to about 3 percent in Fig. 3 and 2 percent in Fig. 4. An investigation of the possible error introduced by Δv, employed in the Fresnel theory, and obtained from the dimensions of the apparatus, shows that at maxima and minima a change of 0.4 percent in the value of Δv can change the intensity of the theoretical curve by as much as 2 percent. The magnitude of this error is not constant throughout a given pattern but fluctuates as Δv takes different positions on the spiral and becomes of greater importance for large values of Δv. For Fig. 3 Δv was 4.34 obtained with an accuracy of about 0.4 percent. For this particular value of Δv known to within 0.4 percent, the maxima and minima in the resultant theoretical curve are known to within only 2 percent. For Fig. 4, where Δv was 5.42, obtained with the same accuracy, the effect is not so marked and amounts to only 1 percent. However, this might explain a part of the disagreement observed in the intensity ratios. No large deviations from the calculated curves are observed in any of the patterns and such deviations as do occur could apparently be corrected by a refinement of the experimental technique. It does not seem unreasonable to conclude, therefore, that within the limits of experimental error, the single-slit diffraction patterns as observed and as calculated by the Fresnel theory are identical.

Calculation of the Luminous Efficiency of Ionized Cesium Vapor

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(Received January 20, 1939)

Theoretical computations are given for the radiation and electrical characteristics of the cesium discharge for pressure and power input above the range of present experimental technique. For a given pressure and temperature the electron concentration is given by Saha's equation, the resistivity depends on the temperature and electron concentration and for sufficiently high pressures the power input is not much greater than the total power radiated by the vapor. The power radiated is evaluated from values of the emissivity times the radiant flux of a blackbody at the vapor temperature. Graphical integration gives the total radiant and luminous flux. An optimum luminous efficiency of about 180 lumens per watt is estimated for a fairly wide range of conditions centering around tube diameter 1 cm, pressure 176 mm, and temperature 6000°K. This requires a current of about 300 amperes and a gradient of 22 volts per cm. The brightness temperature is 5400°K and the color temperature roughly 3700°K. At 760 mm the brightness and color are nearly those of a blackbody. Values for various conditions are tabulated.

INTRODUCTION

THE ideal source of illumination is here considered to be one giving a spectral distribution in the visible range comparable with that of a blackbody at 6000°K and having a high luminous efficiency. The continuous recombination spectrum of an ionized gas deserves serious consideration as a possible means of realizing such a source. A recent paper[1] gives a study of the continuous spectrum emitted by cesium vapor with current densities sufficient to give nearly complete ionization at vapor pressures up to 17 mm of mercury. The brightness increases as the square of the pressure but even at the highest pressure used the luminous efficiency is somewhat inferior to that of an incandescent lamp under normal operating conditions. The discharge was operated in a quartz capillary tube and chemical reaction of cesium and silica sets a limit to the operating pressure.

It may be of doubtful value to compute the characteristics of a discharge for conditions that have not yet been realized experimentally but two considerations lend some interest to the investigation. Cesium, because of its low ionization potential and because of the spectral distribution of its recombination spectrum, is probably the most efficient radiator of all atoms insofar as the continuous radiation is concerned. Furthermore, the experimental data for cesium give a better basis for the computations than in any other case.

It has been found that for high current densities and for pressures above 0.3 mm the gas temperature and electron concentration are nearly in equilibrium with the electron temperature so that the electron concentration is given by Saha's equation

$$\log \frac{N_e^2}{N_1} = 15.385 + \tfrac{3}{2} \log T - 0.434 \frac{E_i}{kT}, \quad (1)$$

where N_e is the number of electrons and ions per cm^3, N_1 is the number of atoms in the normal state and E_i is the ionization energy.

For ionization in excess of about 30 percent the resistivity of a discharge has approximately the theoretical value for a completely singly ionized gas and depends only on electron concentration and temperature.[2]

$$X/I = \frac{1.48 \times 10^4}{T^{\frac{1}{2}}} \log \frac{0.72 \times 10^6 T^2}{N_e^{\frac{1}{2}}}, \quad (2)$$

where X is the gradient in volts per cm and I the current density in amperes per cm^2.

The power input per cm tube length is equal to the power radiated by the gas plus the power dissipated at the tube walls by wall recombination. The latter is predominant at pressures below 0.3 mm where measurements are available,

[1] F. L. Mohler, Nat. Bur. Stand. J. Research **21**, 697 (1938), RP 1150.

[2] F. L. Mohler, Nat. Bur. Stand. J. Research, **21**, 873 (1938), RP 1162. Gvosdover, Physik. Zeits. Sowjetunion **12**, 164 (1937).

but since the wall loss increases slowly with the pressure while the power radiated increases as the square of the pressure, at sufficiently high pressures the power radiated will become important. For such conditions one has as an approximation;

$$\frac{\pi d^2}{4} XI = \pi d W, \tag{3}$$

where d is the tube diameter and W is the radiant emittance in watts per cm². ($\pi d W$ is the radiant flux per unit tube length.) It is possible to evaluate W for cesium by a graphical integration of the spectral distribution curve. By Eqs. (1), (2) and (3) the electrical conditions required to maintain the temperature T at a pressure p in a tube of diameter d are specified. It remains to be seen whether a high luminous efficiency can be obtained under conditions where the approximations used in Eqs. (2) and (3) are valid and to see what the quality of the radiation will be.

The computation of W is somewhat laborious and it is convenient first to consider the general requirements for high luminous efficiency. For the range of pressure investigated experimentally the luminous efficiency at a given pressure reaches a flat maximum at a current density and temperature sufficient to give about 50 percent ionization. For a constant high temperature and low pressure the luminous emittance increases as the square of the pressure. If the temperature is sufficiently high this increase will continue with increasing pressure until the opacity of the ionized gas becomes appreciable when the luminous emittance approaches that of a blackbody at the temperature T. The advantage of cesium as a radiator is that its opacity and emissivity have a maximum in the visible spectrum but this advantage is lost as it approaches complete opacity. With increasing pressure the ultraviolet and infra-red flux will continue to increase after the luminous flux has reached a nearly constant value. Thus the luminous efficiency will have a maximum value at some pressure such that ad is of the order of magnitude of unity where d is tube diameter and a is the absorption coefficient per cm.

Total Radiant Flux

By the principle of detailed balance one can evaluate the radiant flux by summing the recombination into all levels or by considering the flux at each frequency as $e_\nu J(T, \nu)$ where $e_\nu = 1 - e^{-ad}$, is the emissivity and $J(T, \nu)$ is the blackbody radiant flux at a temperature T. The second procedure is chosen because it avoids some apparent difficulties.

The recombination spectrum consists of a series of bands extending to higher frequencies from each series limit and the absorption coefficient of each band is proportional roughly to ν^{-3} (a variation as ν^{-4} was found for the $6P$ band of cesium). The absorption per cm at a frequency ν is the sum of the absorption of all bands with limits less than ν

$$a_\nu = \Sigma N_n \alpha_{n,\nu} \tag{4}$$

where N_n is the population of atoms in the nth state and $\alpha_{n,\nu}$ is the atomic absorption coefficient of this state. The visible continuous spectrum comes almost entirely from the bands beyond the two limits $6P$ and $5D$ at 5000A and 6000A and the experimental values of the atomic absorption coefficients at the limits are, respectively,[3] $\alpha_2 = 3.6 \times 10^{-17}$ and $\alpha_3 = 3.2 \times 10^{-17}$. (Subscripts 2 and 3 correspond to the hydrogenic notation used below.) The contribution of the $6S$ band in the ultraviolet is entirely negligible. The infra-red bands can be assumed to have hydrogenic values and this absorption is nearly all in the single series of limits 4_3, 5_4, 6_5, etc. The formula of Gaunt[4] is used to evaluate this

$$\alpha_{n,\bar{\nu}} = (1.04 \times 10^{-2})/n^5 \bar{\nu}^3, \tag{5}$$

where $\bar{\nu}$ is wave number in cm^{-1}. The absorption per cm becomes

$$a_{\bar{\nu}} = \Sigma \frac{1.04 \times 10^{-2} N_1 e^{-E_n/kT}}{n^3 \bar{\nu}^3}, \tag{6}$$

where N_1 is the number of atoms in the normal state and E_n the energy of the nth state. To this has been added the absorption due to free-free transitions as given by Gaunt

$$a_0 = (6.7 \times 10^{-18} N_e^2)/\bar{\nu}^3 v, \tag{7}$$

where v is the electron velocity ($6.24 \times 10^5 T^{\frac{1}{2}}$).

These equations give the continuous absorption of isolated atoms but, in an intense dis-

[3] F. L. Mohler, Nat. Bur. Stand. J. Research **10**, 771 (1933), RP 565.

[4] J. A. Gaunt, Proc. Roy. Soc. **A126**, 654 (1929).

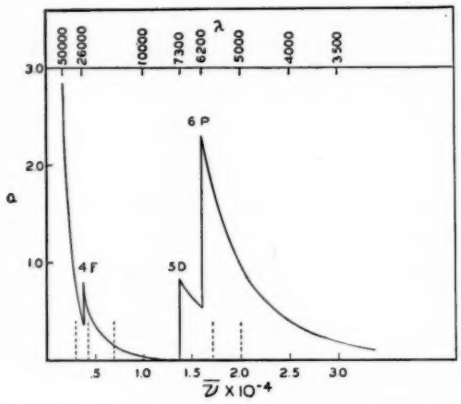

FIG. 1. Absorption of 1 cm of cesium at 5200°K and 176-mm pressure *versus* wave number. Broken lines indicate theoretical limits while peaks are displaced by line broadening.

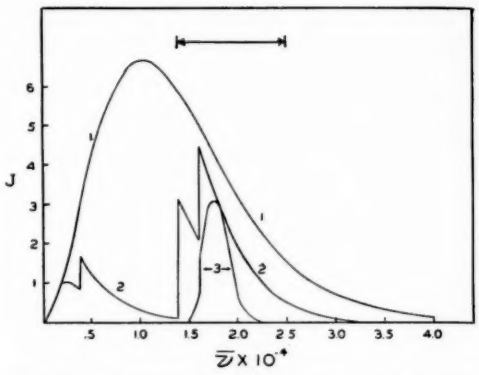

FIG. 2. Radiant emittance (arbitrary scale) at 5200°K as a function of wave number. (1) Blackbody; (2) 1-cm depth of cesium at 176-mm pressure; (3) curve 2 multiplied by the relative luminosity function. Double arrow shows visible spectrum range.

charge, lines are broadened and merge into a continuous spectrum at frequencies less than the theoretical limit. From the last resolved line to the limit the absorption and emission follow the same law as beyond the limit.[5] The relation between the quantum number n_m of the last resolved line, and the electron concentration is known for the D and F series of cesium.[6]

$$\log n_m = 3.13 - 0.137 \log N_e. \tag{8}$$

It is here assumed that all continua begin at a frequency $\tilde{\nu}_0 - R/n_m^2$ where R is the Rydberg constant rather than at $\tilde{\nu}_0$. This greatly increases the calculated radiant flux at high electron concentrations.

Line emission can be neglected in comparison with the continuous spectrum except for the two resonance lines near 8500 and 8900A. This radiant emittance is assumed equal to that of a blackbody at a temperature T over a width $\Delta\tilde{\nu}$ where

$$\Delta\tilde{\nu} = 4.7 \times 10^{-15} N_1 d^{\frac{1}{2}} \tag{9}$$

where d is the tube diameter. This relation is based on low pressure experiments[7] and computed values for high pressures involve considerable uncertainty.

[5] F. Mohler, Nat. Bur. Stand. J. Research **17**, 849 (1936).
[6] F. Mohler, Phys. Rev. **54**, 313 (1938).
[7] F. Mohler, Nat. Bur. Stand. J. Research **9**, 493 (1932), RP 485.

RESULTS

Figure 1 is a plot of the absorption of 1 cm of vapor at 176-mm pressure and 5200°K. For this case there is 50 percent ionization and lines beyond $n=6$ are merged together so that the continuous bands extend 3000 cm^{-1} beyond the theoretical limits. Actually the low frequency edges of the bands are rounded off but the curves are drawn to correspond to the simplified assumptions. About half of the absorption for $\tilde{\nu}$ less than 3000 cm^{-1} comes from free-free transitions if it is assumed that levels from n_m to infinity exist but overlap. If it is assumed that they do not exist then the ionization potential is lowered and N_e increased enough to account for all this absorption.

Curve 1 of Fig. 2 is the blackbody emittance at 5200°K on a wave number scale. Curve 2 is the emittance of the cesium continuous spectrum for the conditions of Fig. 1 and the area under the curve is proportional to the emittance in watts per cm^2. To this must be added the contribution of the resonance lines which is for this case about 13 percent of the continuous spectrum. Curve 3 is curve 2 times the relative luminosity function and the area under it is proportional to the luminous emittance in "light watts" per cm^2. The ratio of this area to the total area is the luminous efficiency of the total cesium emittance relative to that at 5550A. Curves 2 and 3 show that the cesium emission is very favorable for high luminous efficiency.

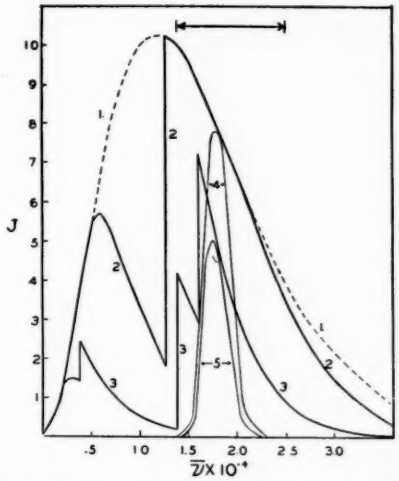

FIG. 3. Radiant emittance at 6000°K. (1) Blackbody; (2) 1-cm depth of cesium at 760 mm; (3) 1 cm of cesium at 176-mm pressure; (4) curves 1 and 2 times relative luminosity; (5) curve 3 times relative luminosity. Double arrow shows visible spectrum range.

Figure 3 gives: (1) the blackbody emittance at 6000°, (2) the emittance of 1 cm depth of cesium at 760-mm pressure and 6000°, (3) the emittance of 1-cm depth of cesium at 176-mm pressure and 6000°. Curves 4 and 5 are the luminous emittance curves derived from 2 and 3. At the higher pressure the emittance in the visible range is practically that of a blackbody though the total radiant emittance is appreciably less. The curves illustrate the reason why the luminous efficiency decreases with increasing pressure.

Table 1 summarizes the computations which have been made for conditions defined in the first 3 columns. Relative efficiencies give the ratio of "light watts" to total watts radiated and the over-all efficiency is somewhat less. A long extrapolation of experiments on power input indicates that at 176-mm pressure in a 0.1-cm tube the wall loss will be of the same magnitude as the radiation. For the other cases it will be a tenth as much. It is concluded that the actual efficiency is within 10 percent of the tabulated value except for the case noted and that Eq. (3) can be used in computations of X and i with the same exceptions. Actual values of X and i for the 0.1-cm tube at 176-mm pressure will be greater than the tabulated values by a factor of the order of $\sqrt{2}$.

Figure 4 gives plots of the luminous emittance and relative efficiency as a function of tempera-

TABLE I. *Radiation and electrical characteristics of ionized cesium.*

T	p* MM	d CM	N_+/N_1*	Radiant Flux Watts Per Cm Tube Length		Relative Efficiency, "Light Watts" Per Watt		X/I OHMS-CM	X VOLTS/CM	i AMP.
				Black	Cs	Black	Cs			
4000	176	1	0.142	4.6×10^3	1.26×10^3	0.078	0.22	0.13	14.5	87
		0.1		4.6×10^2	26		.19		>20.7	>1.26
4500	176	1	.35	7.34×10^3	2.8×10^3	.10	.24	.088	17.8	158
		0.1		7.34×10^2	56		.23		>25	>2.24
5200	176	1	1.0	13.2×10^3	4.6×10^3	.12	.27	.068	19.8	232
		0.1		13.2×10^2	82		.27		>26.7	>3.07
6000	176	1	3.1	23.3×10^3	6.7×10^3	.135	.29	.058	22.3	300
		0.1		23.3×10^2	104		.30		>27.8	>3.75
7000	176	1	9.2	43.2×10^3	7.8×10^3	.135	.30	.050	22.3	350
		0.1		43.2×10^2	107		.29		>26	>4.10
5200	760	1	.55	13.2×10^3	11.9×10^3	.12	.133	.059	30	400
		0.1		13.2×10^2	620		.14		68.4	9.05
6000	760	1	1.0	23.3×10^3	19×10^3	.135	.16	.050	34.8	545
		0.1		23.3×10^2	800		.21		71.5	11.2
7000	760	1	3.2	43.2×10^3	35×10^3	.135	.16	.041	43	815
		0.1		43.2×10^2	1080		.23		75	14.4

* For convenience electron pressure is not included in p which is the partial pressure of ions plus atoms and N_1 is the total number of atoms uncorrected for the fraction in excited states.

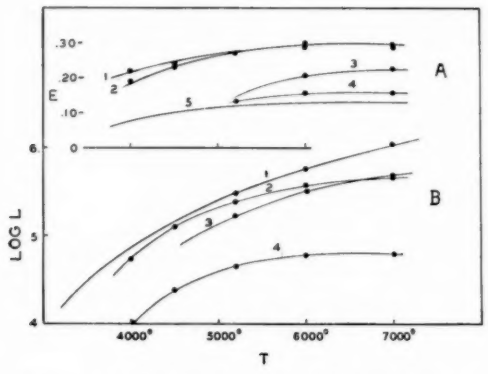

FIG. 4. *A*, efficiency in "light watts" per watt as a function of temperature. (1) 1-cm depth of cesium at 176 mm; (2) 0.1 cm of cesium at 176 mm; (3) 0.1 cm at 760 mm; (4) 1 cm at 760 mm; (5) blackbody. *B*, Log of lumens per cm² *versus* temperature. (1) Blackbody (1 cm of cesium at 760 mm indicated by dots); (2) 1 cm of cesium at 176 mm; (3) 0.1 cm of cesium at 760 mm; (4) 0.1 cm of cesium at 176 mm.

ture. The luminous emittance of a 1-cm diameter tube at 760-mm pressure is practically that of a blackbody (curve *B*1) between 5200°K and 7000°K but will be less than that of a blackbody at lower and higher temperatures. At 176-mm pressure the luminous emittance (curves *B*2 and 4) has a maximum value at about 7000°K. The relative efficiencies at 176-mm pressure (curves *A*1 and 2) remain essentially constant above 4500°K but, as noted before, the over-all efficiency of the 1-mm tube may be as low as half the plotted value. It is concluded that nearly maximum efficiency is attained over a wide range of conditions where the absorption coefficient times the thickness is of the order of unity and the temperature sufficient to give over 50 percent ionization.

Table II gives further computational results on the radiation for the conditions of Table I. The brightness is conveniently expressed in terms of the temperature T_B of a blackbody which matches the cesium spectrum in brightness at 5600A. The cesium discharge probably does not have a true color temperature except when it is opaque but the emittance ratio between 6200A and 5000A is a definite index of the color. The temperature T_c of a blackbody giving the same emittance ratio is given in the last column. This temperature is much less than that of the cesium unless the gas is nearly opaque. For a nearly transparent layer T_c increases slowly at high temperatures and would be about 3700°K for a cesium temperature of 10,000°K. It is of interest that, for a 1-cm tube at 176-cm pressure, T_c has a nearly constant value of 3700°K for temperatures above 4533°. Evidently one cannot approximate the color temperature of 6000°K without exceeding the range of opacity which gives the best

TABLE II. *Luminous characteristics of cesium radiation.*

T	p	α	LUMENS/CM²	LUMENS PER WATT	T_B 5600A	T_C 6200A TO 5000A
4000	176	1 cm	5.5×10^4	137	3800°K	3120°K
		0.1	0.98×10^4	<118	3020°	2540°
4500	176	1	13.3×10^4	150	4300°	
		0.1	2.5×10^4	<143	3300°	2700°
5200	176	1	24.6×10^4	168	4920°	3800°
		0.1	4.4×10^4	<168	3680°	2920°
6000	176	1	38.4×10^4	180	5400°	3700°
		0.1	6.2×10^4	<187	3840°	3100°
7000	176	1	46×10^4	187	5550°	3700°
		0.1	6.2×10^4	<180	3840°	3220°
5200	760	1	31.4×10^4	83	5200°	5200°
		0.1	17×10^4	87	4650°	3370°
6000	760	1	60×10^4	100	6000°	6000°
		0.1	33×10^4	131	5200°	3600°
7000	760	1	110×10^4	100	7000°	7000°
		0.1	49×10^4	143	5600°	3700°

efficiency. In the range of high efficiency T_e will be roughly 4000°K and T_B will be about 5000°K.

Conclusion

It is beyond the scope of this paper to consider the problem of realizing these conditions experimentally. Even if it is impossible or impractical to run a cesium discharge at such high currents and pressures the computations are of some general interest. In a very intense discharge most of the radiation will, in general, be in the continuous spectrum and these results set a fairly definite upper limit of about 180 lumens per watt to the luminous efficiency. The luminous efficiency of a high pressure mercury arc reaches a nearly constant value of 65 lumens per watt with increasing pressure and power input.[8] It is less than the values for cesium or a blackbody at the same temperature (6000° to 8000°) because the mercury spectrum has maxima in the near infra-red and ultraviolet. The radiation of all atoms becomes identical as the emissivity approaches one. The temperature required will be roughly proportional to the ionization potential, and the power input proportional to the fourth power of the ionization potential. The power required is, in most cases, impossible to maintain in a steady discharge but can be realized in condenser discharges.[9] Another paper[10] gives the relations between temperature, current and gradient for the case when the gas is opaque and the ionization so high that Eq. (2) can be used.

[8] Marden, Beese and Meister, Trans. I. E. S. **33**, 147 (1938).

[9] J. A. Anderson, Astrophys. J. **75**, 394 (1932).

[10] F. Mohler, Nat. Bur. Stand. J. Research, **21**, 873 (1938), RP 1162.

The Spectral Emissivity and the Relation of True Temperatures and Brightness Temperatures of Platinum

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(Received November 29, 1938)

Spectral emissivities of platinum for $\lambda = 660$ mμ have been measured by using an optical pyrometer to compare the brightness of the surface of an incandescent tubular filament with that of a small hole through the wall of the tube. They have been used to compute true temperature as a function of brightness temperature, for the range 1200°K to 1900°K true temperature. The emissivity was found to vary linearly with temperature, from 0.283 at 1200°K to 0.295 at 1900°K. The brightness of an incandescent platinum surface was found to vary with angle of emission rising to a maximum 1.24 times the normal brightness at an emission angle of 80° and decreasing thereafter.

I. INTRODUCTION

THE problem of finding the relation between true temperature and brightness temperature for platinum at incandescent temperatures has never been satisfactorily solved, previous results being very discordant.[1]

This work was undertaken with the purpose of eliminating disturbing influences, and measuring the spectral emissivities of this important metal with a precision greater than has been achieved heretofore.

II. EXPERIMENTAL

Method

A magnifying optical pyrometer was used to measure the brightnesses of incandescent platinum and a blackbody at the same temperature. The platinum was tubular with an outside diameter of one millimeter and an inside diameter of one-half millimeter and a length of 10 cm. It was purchased from Baker and Company, Ltd. of New Jersey. Three small holes 0.006 in. in diameter widely separated were drilled radially through its wall. It was polished with rouge to prevent partial blackening of the surface radiation, and was mounted in an evacuated bulb as the filament of a lamp. The small holes served as blackbodies.

The pyrometer was first matched with a hole and then with a portion of the platinum surface adjacent to the hole. A hole near the middle of the filament where the temperature was uniform for approximately one-half inch in either direction was used. The hole and the surface were both viewed at an angle of approximately 80 degrees from the axis of the tube. This was to insure that the radiation from the hole was properly built up by multiple reflection to blackbody radiation. The ratio of the brightness of the surface to that of the hole is the normal emissivity at a wave-length determined by the red pyrometer screen. Measurements were made at thirty temperatures from 1090°K to 1835°K. The true temperature in each case was obtained from the brightness of the hole. Some measurements of brightness as a function of angle of emission were made at one temperature.

The bulb was highly evacuated by a diffusion pump, the platinum being heated to incandescence during evacuation. It was found necessary to have the vacuum high and the metal outgassed to get satisfactory pyrometer readings. It had been found that the brightness fluctuated constantly when evacuation was accomplished with a "Hyvac" pump unaided.

When the lamp was new a crystalline pattern was observed on the incandescent surface of the filament. The brightnesses of the different crystal faces were considerably different. When the temperature was raised to 1605°K this pattern disappeared in a few minutes. The filament was then aged for about two hours at approximately 1605°K, and all the reported data were obtained after this aging. The crystalline pattern was never seen again.

* Now at the University of Toledo, Toledo, Ohio.
[1] A. G. Worthing, Phys. Rev. **28**, 183 (1926).

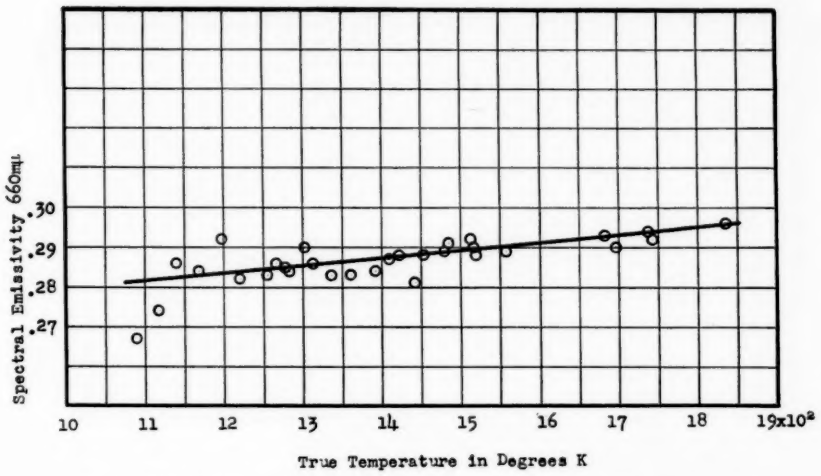

FIG. 1. The spectral emissivity of platinum as a function of true temperature.

III. STANDARDS

The secondary standard of brightness temperature was a hairpin filament tungsten lamp which was calibrated at the palladium point by W. E. Forsythe at Nela Park, Cleveland.

The temperature scale used is based on Wien's law using $C_2 = 14{,}320\mu°K$.[2] On this scale the palladium point is 1828°.

The effective wave-lengths of the red pyrometer screen were also furnished by Dr. Forsythe.

IV. CORRECTIONS

Effective wave-length

The wave-length with which a measured emissivity should be associated is the effective wave-length of the pyrometer screen for blackbody radiations at the true temperature and the color temperature of the platinum.[3] These wave-lengths differ slightly from one temperature to another and it was desired to adjust the emissivities so they would all apply to the same wave-length, 660 mμ. This was done by using the rate of change of emissivity with wave-length determined in some previous work of Worthing.[4] Calculations showed that the corrections were negligible, never greater than 0.0006. They were, however, taken into account.

Difference in temperature between the inside and the outside of the tubular filament

If the interior of the filament is at a higher temperature than the surface, its brightness is greater than that of a blackbody at the temperature of the surface. This means that the uncorrected emissivity is less than the true emissivity. The difference in temperature was computed by means of a formula used by Angell.[5]

$$\Delta T = \frac{i^2\rho}{2k}\left[a^2 \log_e \frac{b}{a} - \frac{b^2 - a^2}{2}\right]$$

where

ΔT = difference in temperature between inside of tube and outside,
i = current density,
ρ = resistivity,
k = thermal conductivity,
a = inside radius,
b = outside radius.

Differences in temperature at 1800°K, 1500°K and 1200°K were 0.24°, 0.12° and 0.05°. Neglect of these differences means errors in the emissivities of 0.0005, 0.0003 and 0.0002 which are smaller than the experimental error.

[2] George K. Burgess, Nat. Bur. Stand. J. Research **1**, 635 (1928).
[3] A. G. Worthing, Phys. Rev. **10**, 184 (1917). W. E Forsythe, Gen. Elec. Rev. p. 749, Sept. (1917).
[4] A. G. Worthing, Phys. Rev. **28**, 182 (1926).
[5] M. F. Angell, Phys. Rev. **33**, 422 (1911).

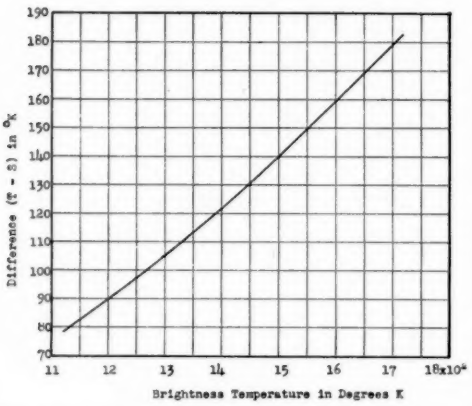

FIG. 2. Difference between true temperature and brightness temperature as a function of brightness temperature.

Lack of blackness of radiation from a hole

If the radiation from a hole in a tube, whose temperature is uniform along its length, is not completely black, the brightness of the hole is less than if completely black, and the computed emissivities will be too large. Assuming a polished cylindrical interior with a reflectivity of 0.75, leads to a brightness within 0.1 percent that of a blackbody for the conditions used in this test (10° angle of emergence). The fact that the author's results for emissivity are lower than those of previous investigators would seem to indicate that blackness conditions were good.

TABLE I. *True temperature, emissivity, and brightness temperature relations for platinum.*

TRUE TEMPERATURE °K	e_λ $\lambda = 660$ mμ	BRIGHTNESS TEMPERATURE °K	TRUE TEMPERATURE MINUS BRIGHTNESS TEMPERATURE $T-S$ °K
1200	0.283	1121.6	78.4
1250	.284	1165.4	84.6
1300	.285	1209.0	91.0
1350	.286	1252.5	97.5
1400	.287	1295.6	104.4
1450	.288	1338.6	111.4
1500	.289	1381.4	118.6
1550	.290	1424.1	125.9
1600	.291	1466.5	133.5
1650	.292	1508.8	141.2
1700	.293	1550.8	149.2
1750	.294	1592.7	157.3
1800	.295	1634.4	165.6
1850	.296	1676.0	174.0
1900	.297	1717.4	182.6

Absorption by glass of lamp bulb

The usual correction for the absorption of the glass of the lamp bulb was made.

V. RESULTS

The emissivities were calculated from the observed brightnesses (Fig. 1). They are somewhat scattered but the trend is definite enough to determine an average line. In locating this line, corrections for effective wave-length variations and the difference in temperature between the inside and the outside of the tube were taken into account. The curve as drawn refers to a wave-length of 660 mμ. The average deviation of the observed values from the line is 0.0017 or about 0.6 percent. Emissivities for temperatures from 1200°K to 1900°K are tabulated in Table I.

The brightness temperatures corresponding to certain true temperatures were calculated from Wien's law

$$\frac{1}{S}=\frac{1}{T}-\frac{\lambda}{C_2}\ln e_\lambda,$$

where T = true temperature, S = brightness temperature, λ = effective wave-length, e_λ = emis-

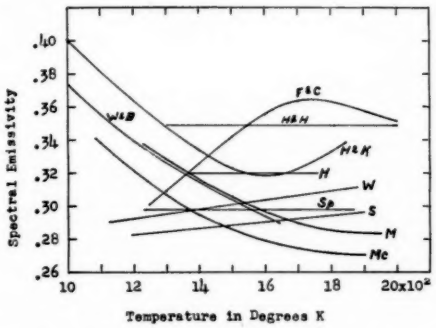

FIG. 3. Published spectral emissivities of platinum at incandescent temperatures for red light as obtained by various experimenters.

CURVE	AUTHOR	YEAR	λ
H and K	Holburn and Kurlbaum	1903	650 mμ
W and B	Waidner and Burgess	1907	660
F and C	Fery and Cheneveau	1909	629
H	Henning	1910	665
M	Mendenhall	1911	658
Mc	McCauley	1913	658
Sp	Spence	1913	658
H and H	Henning and Heuse	1923	647
W	Worthing	1925	665
S	Stephens	1933	660

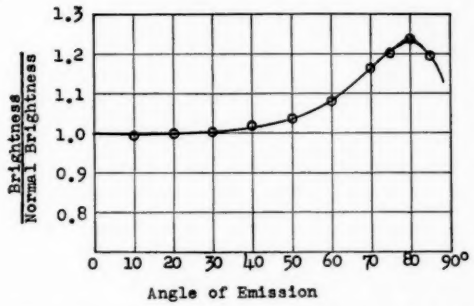

FIG. 4. Deviation from Lambert's law. Ratio of brightness to normal brightness as a function of angle of emission. Platinum at 1392°K.

sivity. These are also tabulated in Table I.[6] The difference between the true temperature and the brightness temperature is plotted as a function of brightness temperature, and is shown in Fig. 2.

A comparison with the results of others is shown in Fig. 3. The emissivities reported in the present work are seen to agree with those of Worthing in variation with temperature but are about 0.01 lower. The final results reported by Worthing are averages of readings by three observers. The agreement in magnitude is excellent if only those results obtained by Worthing himself, the most experienced of the three, are selected.[7] There is about the same agreement in magnitude with Spence's work, but he did not observe any change with temperature. There is little agreement with others. The observed variations in brightness with angle of emission are shown graphically in Fig. 4.

This work was done under the direction of Dr. A. G. Worthing.

[6] It is to be noted that the accuracy of the *relation* between true temperature and brightness temperature depends only upon the accuracy of ϵ_λ.

[7] A. G. Worthing, Phys. Rev. **28**, 182 (1926), Fig. 3. Worthing's observations are shown by circles. The comparison is as follows:

Temp. °K	ϵ_λ Stephens 660 mμ	ϵ_λ Worthing 665 mμ
1343	0.286	0.287
1605	.291	.294
1610	.291	.294
1850	.296	.299

Temperature Corrections in Optical Pyrometry

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(Received January 13, 1939)

THE secondary standards of brightness used to calibrate portable optical pyrometers are usually tungsten ribbons in evacuated bulbs. The tungsten ribbon appears less bright than it actually is because of a loss of light at the two surfaces of the bulb, but this loss is accounted for in the original calibration of the standard. Thus the certificate accompanying the standard will specify that "at 13.00 amperes the filament, viewed through the bulb, will appear to be as bright as a blackbody at 1204° absolute."

A portable pyrometer calibrated against such a standard will indicate correctly when used against a body in a furnace or on a radiating body that is not enclosed in glass, but if the surface being tested is viewed through glass a correction should be made for losses at the two glass surfaces. In some physical experiments the radiating body may lie behind several thicknesses of glass, or it may be necessary to look through the glass at an acute angle, and in these cases the errors in the pyrometer indication may be of considerable size. A recent example of this condition involved an error of 26° when pyrometering a filament at 1500°K, and an uncorrected error of more than 5° was not allowable in the final results.

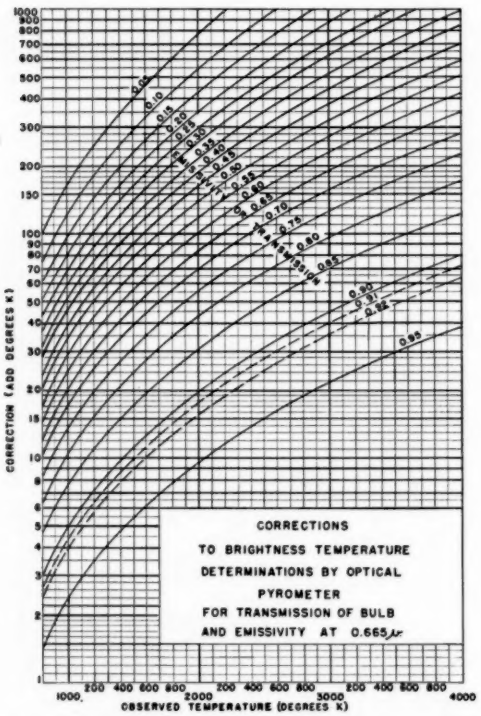

FIG. 1. This chart is constructed so that the errors in pyrometry due to absorption of light by enclosing glass may be determined. Several other types of determinations may be made, as explained in the text.

The accompanying chart, Fig. 1, was constructed to facilitate the making of such glass corrections, and the range of the chart was considerably extended so as to be useful when dealing wth bulbs that are blackened by evaporated metal and transmit only a small part of the incident light.

The temperature scale across the bottom of the chart is, of course, the absolute scale, in degrees Kelvin, and the markings are the uncorrected degrees as indicated by the pyrometer. The curved lines may represent several factors, such as (*a*) transmission factor of glass, (*b*) emissivity of radiating surface, (*c*) combined effects of factors (*a*) and (*b*). Several examples are worked out in the following tabulations to illustrate the various ways in which the chart may be used.

Example 1. Bulb corrections are made as follows:

Observed brightness temperature	2800°K
Bulb transmission (known)	0.91
Bulb correction (from chart)	35°
Corrected brightness temperature	2835°K

Example 2. If the emissivity (at 0.665μ and at the test temperature) is known, the true temperature of the above radiator can be found as follows:

Brightness temperature (using observed temperature scale of chart)	2835°K
Emissivity (known at 0.665μ)	0.60
Emissivity correction (from chart)	204°
True temperature	3039°K

Example 3. From pyrometer readings on the surface of a radiator, and from readings on a cavity in the same radiator the emissivity is determined as follows:

Surface reading	2200°K
Bulb transmission (known)	0.91
Bulb correction	22°
True brightness temperature	2222°K
Cavity brightness (observed)	2550°K
Bulb correction	29°
True cavity brightness temperature	2579°K
Difference, cavity minus surface	357°K
Ordinate at 2222° on temperature scale and abscissae at 357° on correction scale cross near curve marked 0.25 gives emissivity	0.26

Example 4. A filament observed through a clean section of bulb has an observed brightness temperature 45° higher than when observed through a section having a thin metallic coating. The transmission of the coated section can be found with the aid of the chart.

Reading through clear surface	1500°K
Bulb transmission (known)	0.92
Bulb correction	9°
True brightness temperature	1509°K
Reading through coated surface	1360°K
Loss in coated section (1509–1360)	149°K
Curve at 1360° ordinate, 149° abscissae	0.22
Bulb transmission	0.92
Deposit transmission (0.22/0.92)	0.24

TABLE I. *Refractive indices.*

TITLE	INDEX
Clear sealing, glass No. 706	1.482
Clear tungsten sealing, glass No. 705	1.479
Clear heat resisting Nonex, glass No. 772	1.489
Clear heat resisting chemical Pyrex, glass No. 774	1.474
Clear lime (bulb), glass No. 008	1.512
Clear soda lead, glass No. 005	1.546
Clear potash soda lead, glass No. 001	1.542
Clear heat resisting Corex D, glass No. 970	1.478

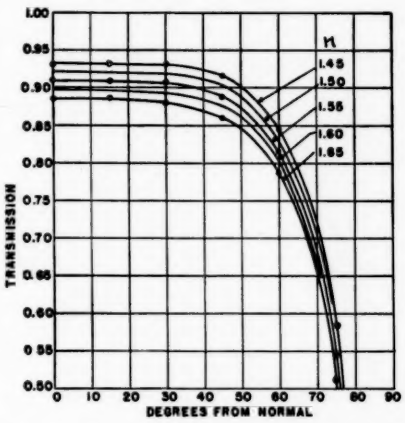

FIG. 2. The loss of light due to reflection at the two surfaces of a piece of glass is here given for indices from 1.45 to 1.65, and for angles of incidence from 0 degree to 75 degrees. No allowance is made for absorption of light within the glass, and this effect if present must be determined in other ways.

Some transmission curves for glass having indices from 1.45 to 1.65 are given in Fig. 2. The transmission is seen to be nearly independent of the angle with the normal up to 40° when the transmission begins to fall off rapidly. The curves indicate the desirability of making the pyrometer readings with the line of sight near the normal to the lamp bulb.

If the glass shows color there will be additional absorption losses that cannot be computed and must be determined by test and by methods that are beyond the scope of the chart.

In Table I are given the refractive indices of some of the Corning glasses that are used for different types of experimental work involving heated filaments.

The Photographic Reciprocity Law at Low Temperature*

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(Received January 14, 1939)

The results of measurements of reciprocity law failure at 20°C and at −186°C are given for two different high speed panchromatic emulsions, one low speed unsensitized emulsion, and one high speed orthochromatic emulsion. These measurements, covering the range in exposure time from $\frac{1}{16}$ second to 17 minutes 4 seconds, show that in each case the reciprocity law failure, at both high and low intensity, is normal at 20°C, but has disappeared entirely at −186°C. These results are in complete agreement with those recently published by Berg and Mendelssohn.

Introduction

In an earlier paper by J. H. Webb and C. H. Evans,[1] results of measurements of reciprocity law failure at the temperature of liquid air (−186°C) were given for two different emulsions. These former measurements showed that the reciprocity law failure had practically disappeared at −186°C. However, there was some indication that there remained at the reduced temperature a small amount of reciprocity law failure at low intensities. As the temperature control in these earlier measurements was not above question, it was thought that the observed drop in the reciprocity curves at the low intensity end might be attributable to a slight heating of the film, caused by radiation through the exposing window during the long times of exposure. Furthermore, measurements carried out by Berg and Mendelssohn[2] (on different emulsions) showed that the reciprocity law failure had completely disappeared at the temperature of liquid air, and this was taken as positive evidence that the measurements in the former paper stood in need of correction. This suspicion of error was mentioned in the former paper, and

* Communication 716 from the Kodak Research Laboratories.

[1] J. H. Webb, and C. H. Evans, J. Opt. Soc. Am. **28**, 249 (1938).

[2] W. F. Berg, and K. Mendelssohn, Proc. Roy. Soc. **A168**, 168 (1938).

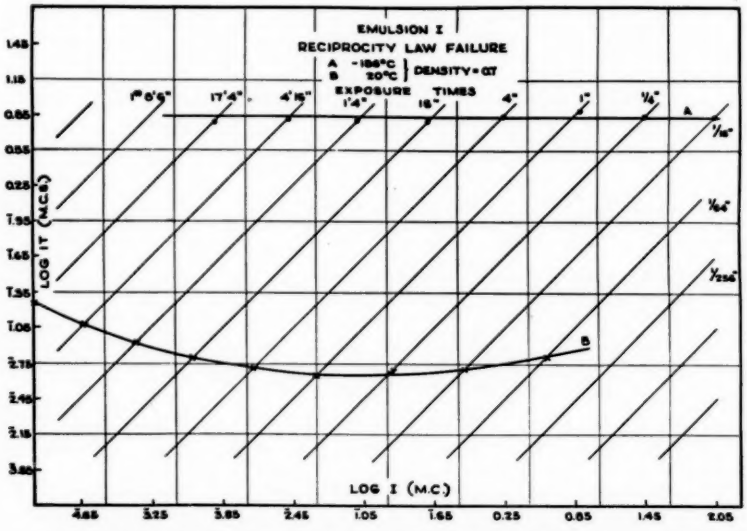

FIG. 1. Effect of low temperature upon the reciprocity characteristic of a high speed panchromatic emulsion.

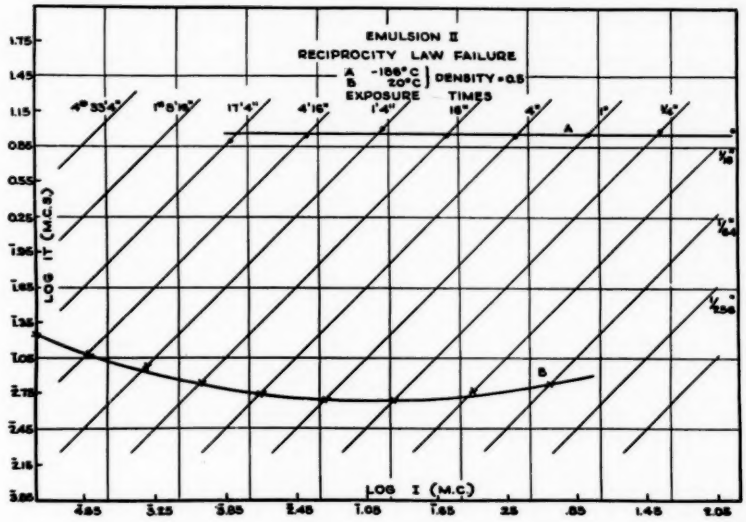

FIG. 2. Effect of low temperature upon the reciprocity characteristic of a high speed panchromatic emulsion.

it was stated at that time that the measurements contained therein were not considered to be discordant with those obtained by Berg and Mendelssohn. New measurements with more accurate temperature control have now been made on four different emulsions including one[3] of those for which reciprocity measurements were presented in the previous paper. These new measurements show that the reciprocity law failure has completely disappeared at −186°C for the entire intensity range covered. Thus the new results obtained here are in complete agreement with the results of Berg and Mendelssohn.

EXPERIMENTAL PROCEDURE

The exposures were made through an intensity-scale step-wedge, and timed by a single revolution of a sector wheel placed just in front of the film. The intensity level for the different exposures was altered by moving the lamp source along a calibrated track, varying the intensity in accordance with the inverse square law. At each new level of intensity, the time of rotation of the sector was altered so as to keep the product (intensity×time) constant. In this way, the factors of intensity and time were reciprocally varied over a range of 1 : 16384. The shortest exposures were $\frac{1}{16}$ second and the longest, 17 minutes 4 seconds. The exposing light was obtained from a tungsten lamp burning at a color temperature of 2750°K. To lower the temperature of the film, the latter was placed behind a neutral density step-wedge held in a metal guide which, in turn, was held vertically against the inside wall of a Dewar flask filled with liquid air. In this way, the emulsion was in direct contact with liquid air, and it was thought, therefore, that there could be no question as to the temperature of the film. The exposing light passed through the double glass wall of the Dewar flask and through a diffusing medium placed directly in front of the step-wedge. It was found desirable to employ this diffusing medium, since otherwise the sharp shadows cast on the film by irregularities in the glass walls of the Dewar flask produced a troublesome pattern in the developed densities. Correction for the density of this diffusing medium has been made in computing the intensity of the exposing light. In the comparison exposures at room temperature, the exposures were made in the same way as that just described except that the flask was not filled with liquid air.

[3] The other emulsion used in the previous paper was not tested here since it is double-coated and shows anomalous behavior on this account.

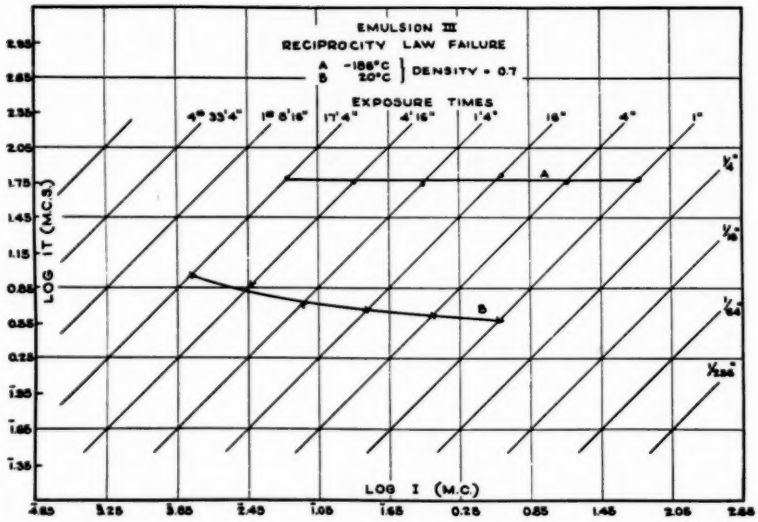

FIG. 3. Effect of low temperature upon the reciprocity characteristic of a low speed unsensitized emulsion.

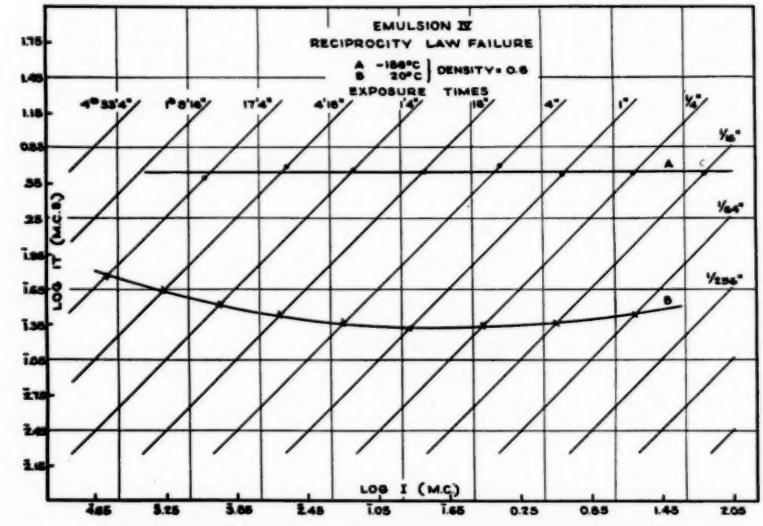

FIG. 4. Effect of low temperature upon the reciprocity characteristic of a high speed orthochromatic emulsion.

Results

The emulsions tested are listed and described below, each being assigned a number, which will be used to designate the emulsion in the graphs to follow:

Emulsion No.	Type
I	High speed panchromatic[4] emulsion
II	High speed panchromatic emulsion
III	Low speed unsensitized[5] emulsion
IV	High speed orthochromatic emulsion.

The results are shown graphically in Figs. 1 to 4. The curves in the figures are the so-called reciprocity law failure curves. Ordinates of these curves correspond to the logarithms of the exposure values required to produce a given constant density, while the abscissae correspond to the logarithms of the intensity levels at which the exposures were made. The exposure times can be ascertained from the diagonal lines. The density for which each curve is drawn is given at the top of the graph. In each case, a reciprocity curve is given for room temperature (20°C) and for liquid-air temperature (−186°C). In all instances,[6] the reciprocity failure curves at 20°C show a normal behavior, that is, a minimum at some intermediate value of intensity and a rise at high and low values of intensity. The curves at −186°C show that the reciprocity law failure has completely disappeared at this temperature for all the emulsions tested.

Since a full discussion of the theory of reciprocity law failure and its variations with temperature was given in a former paper by J. H. Webb and C. H. Evans,[1] as well as in the paper by Berg and Mendelssohn,[2] it is considered unnecessary to enter into this subject again here.

[4] Same as one of the emulsions (II) used for reciprocity tests in the earlier paper by Webb and Evans, reference 1.

[5] Same emulsion as that employed for interrupted and Herschel exposures in previous paper by Webb and Evans, reference 1.

[6] The rise at high intensity for emulsion III was not observed because intensities sufficiently high were not reached. This emulsion is known to exhibit a normal behavior at high intensities.

Summary

Results of measurements of reciprocity law failure on four different emulsions for 20°C and −186°C are presented. It is shown that the reciprocity law failure at both high and low intensity has disappeared at the low temperature. These results are in complete agreement with results recently published by Berg and Mendelssohn.

Acknowledgment

The work described here is a continuation of investigations started by J. H. Webb. The authors wish to thank Dr. Webb for suggesting this research, and for his constant and helpful advice throughout the investigation.

Measurements on Direct and Indirect Adaptation by Means of a Binocular Method

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Changes in the sensitivity of the fovea of one of the eyes (the right eye) under influence of a glaring source of light shining into that eye can be demonstrated by comparing the visual impression obtained by that eye from a dimly illuminated surface with the impression obtained by the other eye (the left eye) from a similar surface. By adjusting the illumination of one of the surfaces the apparent brightnesses can be made to match exactly. In this way a quantitative method of measuring the sensitivity of the fovea under influence of various conditions of illumination is obtained. When a small area of the retina is illuminated the sensitivity of this area and also of the surroundings drops to a considerably lower level within 0.1 second (α-adaptation). The rate of recovery of the sensitivity after exposure is strongly dependent upon the time of exposure (β-adaptation). α-adaptation can neither be accounted for by assuming a strong adaptive influence of the stray light within the eye, nor by assuming a lateral diffusion of photosensitive substances or of photolytic products over the retina. The hypothesis is put forward that α-adaptation is of electrical origin. This hypothesis accounts for many of the phenomena observed with incandescent lamp light but not for those observed with strongly chromatic light. The only hypothesis presenting itself at present to account for these phenomena seems to be that α-adaptation has its seat in the retinal synapses.

INTRODUCTION

IT is possible to observe any difference of sensitivity, which may exist between the two eyes by comparing alternately the visual impressions which each eye receives from a given test object. A critical method of demonstrating this difference consists in arranging two identical objects against a dark background together with suitable screens in such a way that either eye perceives only one of the objects. Differences in sensitivity between the two eyes then manifest themselves in a different brightness or color perception of the fields formed by the two objects. If the illumination of one of the objects can be varied until the impressions again match, the difference in sensitivity between the two eyes may be expressed in measurable physical quantities. Thus a quantitative method of measuring adaptation is obtained.

This binocular method, which following Hering[1] was used by Inouye and Oinuma[2] and by Dittler and Koïke,[3] has been utilized anew by Wright[4, 5] with such refinements of experimental technique that a new stage in the study of adaptive phenomena has been reached. It may be mentioned in this respect that independently von Békésy[6] developed a binaural method for measuring adaptative phenomena in the ear, the results of which bear many interesting resemblances to those obtained with the eye.

The adaptation of the fovea, with which the present paper will deal chiefly, may be divided into *direct adaptation* and *indirect adaptation* according to whether or not the changes in sensitivity of the fovea are brought about by a light source occupying the same place in the field of view as the test object. The investigations of Wright exclusively dealt with direct adaptation.

The experiments[7, 8] to be described in the present paper deal with the more general question of investigating the ways in which the light perception of the fovea may be affected by the

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[1] E. Hering, "Grundzüge der Lehre vom Lichtsinn," *Graefe Saemisch Handbuch Ges. Aug.* (1905).

[2] N. Inouye and S. Oinuma, "Untersuchung der Dunkeladaptation des einen Auges mit Hilfe des helladaptierten Andern," Gr. Arch. Ophth. **79**, 145–159 (1911).

[3] R. Dittler and I. Koïke, "Ueber die Adaptationsfähigkeit der Fovea Centralis," Zeits. Psych. Physiol. Sinnesorg, **46**, 166–178 (1912).

[4] W. D. Wright, "The measurement and analysis of colour adaptation phenomena," Proc. Roy. Soc. **B115**, 49–87 (1934).

[5] W. D. Wright, "The foveal light adaptation process," Proc. Roy. Soc. **B122**, 220–245 (1937).

[6] G. von Békésy, "Ueber die Bestimmung des einem reinen Tonempfinden entsprechenden Erregungsgebietes der Basilarmembran vermittelst Ermüdungserscheinungen," Physik. Zeits. **30**, 115–125 (1929).

[7] L. S. Ornstein and J. F. Schouten, "De rol van electrische, photochemische en diffusieverschynselen by het zien." Proc. K. Akad. Amsterdam **40**, 376–382 (1937).

[8] J. F. Schouten, "Visueele meting van adaptatie en van de wederzydsche beinvloeding van netvlieselementen," Ac. Thesis (Utrecht, 1937).

presence of a glaring light source somewhere in the field of view.

The principal possibilities by which the foveal light perception may be affected are (Fig. 1):[9]

I. Change in sensitivity of the fovea under influence of that part of the retina which corresponds with the image of the glare source.

II. Re-illumination of the fundus by the image of the glare source.

III. Diffraction and scattering in the optic media.

IV. Change in sensitivity of the fovea under the influence of that peripheral part of the retina which corresponds with the illuminated transparent part of the sclerotic.

V. Re-illumination of the fundus by the light passing the sclerotic.

VI. Change in sensitivity of the fovea under influence of the strary light within the eye falling on the fovea.

The two forms of re-illumination (II, V) can be easily perceived entoptically. Diffraction and scattering (III) have received considerable attention ever since the first work of Holladay[10] and have been investigated thoroughly anew in the recent work of le Grand.[11] The influences I and IV transmitted by the retina are still of a rather hypothetical nature. Although evidence of the existence of such an effect is provided by some aspects of simultaneous contrast, by summative effects in electrophysiology[18] and by the existence of lateral cells in the retina, we are still ignorant of the exact nature of that effect and more especially of its relative importance in visiodynamical phenomena.

As regards their effect upon the light perception the possibilities I to VI must be divided into two fundamentally different groups. The light perception of the fovea may be altered either because (a) the sensitivity of the fovea and thus the brightness of the object is lowered (possibilities I, IV and VI) or because (b) an extra veil of light is thrown on the image of the object as well as on its surroundings, whereby the brightness of the object as well as of its surroundings is heightened (possibilities II, III and V).

Both the veiling and the adaptive type of influence have the same effect upon the threshold of visibility of the object, as in both cases the contrast between the object and its surroundings is lowered. Thus the essential difference between these two entirely different causes is never disclosed except in a secondary manner when employing threshold-measurements such as are mostly used in experiments on glare. The binocular method, however, which allows the brightness itself (not merely the contrast) to be measured opens the way to distinguish clearly between these two possibilities.

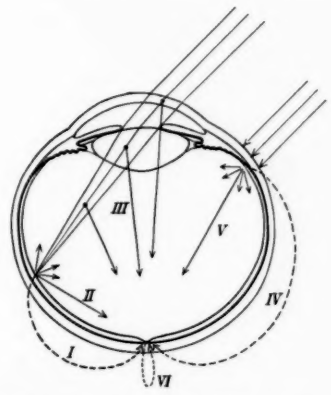

FIG. 1. Schematic representation of the different ways in which the perception of light by the fovea may be affected by the presence of a glaring light source.

APPARATUS AND EXPERIMENTAL TECHNIQUE

The apparatus was constructed in such a way that all observations, adjustments and readings could be made by one experimenter (J. F. S.). It might appear that an undesirable subjective element is thereby introduced into the measurements. Many of the phenomena encountered in these experiments, however, were complicated and thus had to be noticed and ascertained before a reliable method of quantitative measurement could be developed. Even then it was still necessary to take note of seemingly insignificant subjective phenomena which were often found to play an essential rôle. This method therefore seemed preferable to start with.

[9] J. F. Schouten, "Zur Analyse der Blendung," Proc. K. Akad. W. Amsterdam **37**, 506–516 (1934).

[10] L. L. Holladay, "The Fundamentals of Glare and Visibility," J. Opt. Soc. Am. **12**, 271–319 (1926); **14**, 1–15 (1927).

[11] Y. le Grand, "Recherches sur la diffusion de la lumière dans l'oeil humain," Rev. d'optique **16**, 201–214 241–266 (1937).

At a distance of 80 cm from the observer's eyes E (Fig. 2), two pieces of paper, mostly white, of semicircular form and 2.9° in diameter are placed together with the black screens M_1 and M_2 in such a way that the right eye can only perceive the *test object* T, the left eye only the *object of comparison* C. The object of comparison C is illuminated with constant illumination by lamp A, the test object T is illuminated variably by lamp B which can be moved along rail R_1.

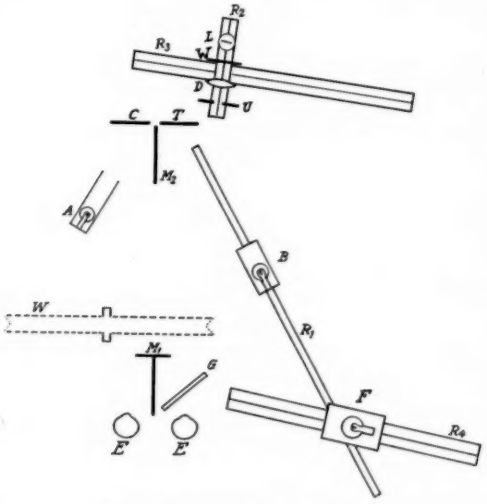

FIG. 2. Sketch of the apparatus.

Rail R_2 carrying the glare source L can be moved along rail R_3. Rail R_2 carries, furthermore, a photometric wedge W, a lens D and a shutter U. With this arrangement an illumination up to 200 lux can be obtained at the observers right eye, the left eye being shielded by screen M_1. The glare source is seen on the same horizontal line as the objects (Fig. 3). The glare angle can be varied from 2° to 25°. Rail R_1 makes an angle of 20° with the horizontal plane so that lamp B cannot obscure the glare source L.

On a horizontal rail R_4 lamp F is mounted at the same height as the observers eyes. This lamp was used, when necessary, together with a piece of plain glass G to obtain direct adaptation. The use of the slit-wheel W will be described in a later paragraph.

The shutter U as well as the switches for the various light sources can be operated by the observer. The experimental room was dimly illuminated, the observer was not subjected to any preliminary dark adaptation.

The experiments were carried out as follows; The observer placed his head in position before screen M_1 and matched the brightness of the surfaces by adjusting lamp B. The difficulties which have already been encountered in monocular photometry as regards the dividing line between the two halves of a photometric field present themselves in far greater measure in binocular photometry. A special difficulty arises with inexperienced observers because of the wandering convergence of the eye-axes. This manifests itself in a seemingly different relative position of the two objects in the field of view. After some experience, however, this convergence comes under conscious control so that the two halves of the photometric field can be made to join exactly. The accuracy of one reading is about seven percent. The reading having been taken, shutter U was opened and a second adjustment made. Care was taken in routine measurements always to fixate the exact center of the circular field formed by the two objects. By this method it is possible to match the two semi-circular fields formed by the objects with reference to brightness only. In Wright's experiments the comparison field was formed by the prism of a

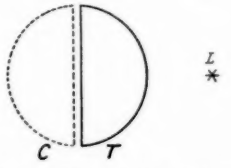

FIG. 3. Relative position of the glaring light source L, the test object T and the object of comparison C. L and T are only visible to the right eye, whereas C is only visible to the left eye.

trichromatic colorimeter. The observer was thus enabled to make a perfect, complete color match. The former method was adopted here, especially because it allowed great rapidity of measurement. It was thus rendered possible to take measurements every 15 seconds with ease and every five seconds if necessary, which proved especially useful when studying the recovery of the sensitivity after exposure. Though the trichromatic method is of course the more exact way to study

the differences between the two eyes especially as regards the dependence of the sensitivity upon the wave-length, it has the disadvantage of being slower. For studying the dynamics of the phenomena with incandescent-lamp light the method followed here seems preferable and of sufficient accuracy to justify the conclusions drawn.

Proof of the Existence of an Adaptive Influence

If the objects are matched and shutter U is opened, so that the glare source L becomes visible to the right eye, the test field is suddenly seen to be much darker than the comparison field. After short times of exposure the brightness quickly returns to its original value; after longer times this recovery takes longer, until, after more than five minutes exposure, a final stage as judged by the maximal time of recovery, also about five minutes, seems to be reached.

Before being justified in interpreting this effect of drop in brightness as a drop in sensitivity, it had to be ascertained whether changes in the diameter of the pupil could not account for the observed effects. If, for example, the narrowing of the pupil of the illuminated eye were greater than in the other eye, effects of the type described above could be expected.

This was investigated by means of a pupillometer inspired by Moss.[12] Instead of two small holes of variable distance, use was made of two silver grains one mm in diameter and illuminated by a point source of light from above. This pupillometer has the great advantage that it obscures practically no part of the field of view. Thus the diameter of the pupils could be measured during and after exposure.

It was ascertained that for the observer's eyes when one eye was illuminated, both pupils change to the same extent and with the same dependence upon the time. Moreover both this extent and the dependence upon the time are of quite a different order than those observed for the brightness of the test field. The brightness of the test field for example drops immediately without any noticeable time lag; whereas, the contraction of the pupils only sets in after a time lag of about 0.2 second.

[12] F. K. Moss, "A Modified Broca Pupillometer," J. Opt. Soc. Am. **22**, 735–738 (1932).

We are therefore justified in interpreting the observed drop in brightness as a drop in sensitivity of the exposed eye relative to that of the unexposed eye. This immediately leads to the important conclusion that the main influence of an extraneous light source on the visual perception of the fovea is not that of an extra veil of light (II, III or V), but that of an adaptive phenomenon (I, IV or VI). Had the former alternative been the case it would have manifested itself in a higher brightness of the test field.

The next question to be answered is, whether the difference in sensitivity found between the two eyes may be interpreted as a lowered sensitivity of the exposed eye alone. According to Wright changes in the unexposed eye take place but to a negligible extent. To investigate the effects produced in the unexposed eye, the threshold of visibility of the left eye was determined before and after very strong illumination of the right eye. The threshold was found to be 60 percent higher in the second case. This, however, could be accounted for entirely by the remaining contraction of the pupil which now of course comes into play. Secondly, the right eye was strongly illuminated with spectrally homogeneous light resulting in a strong discoloration of objects seen by the right eye. The left eye, however, did not show any appreciable change in color perception even for colors of very low saturation.

In the region of intensities investigated here, we may, therefore, interpret the differences in sensitivity found between the two eyes as changes in sensitivity of the exposed eye alone. The binocular compensation method thus enables us to calibrate the one exposed, changed eye in terms of the other unexposed, unchanged, standard eye.

During exposure, lamp B is moved forward until again a match is obtained. If E_1 be the illumination of the test object before, E_2 during exposure, we may interpret the quantity

$$\rho = E_1/E_2 \qquad (1)$$

as the *relative sensitivity* ρ of the exposed eye. Values of ρ ranging between 1 and 0.04 were obtained in this way. Sometimes it proved more convenient to express changes in sensitivity by the relative increase of illumination needed for

the two matches. This quantity

$$V=(E_2-E_1)/E_1=1/\rho-1 \qquad (2)$$

will be called the *glare index* V. When ρ changes from 1 to 0.04, V changes from 0 to 24.

It is not at all sure beforehand that the sensitivity ρ, defined in this way, is a direct measure of any fundamental property of the retina. The ultimate brightness perception of the test field should be considered as a temporary equilibrium between excitative and inhibitory processes in the retina which are brought about as well by the test field itself as by the extraneous light source. Thus ρ might vary with the brightness of the test field. This indeed is reported by earlier investigators. The independence of ρ upon the brightness of the viewed field, supposed by von Kries[13] in the "*law of coefficients*" was found by Wright to hold true for direct adaptation. In the present investigations the law of coefficients was found to hold true for direct as well as for indirect adaptation in the brightness range used for the comparison field (0.3−5 c/m²). ρ was also found to be independent of the size of the fields (0.7°×0.7°−3.6°×7.2°).

It should be stated, contrary to Wright's conclusion, that the validity of the law of coefficients provides no evidence whatever for the existence of a linear relation between original stimulus and ultimate response. Measurements of this kind, which relate the brightnesses that are necessary to produce the *same* visual perception can never supply information on the relation between *different* visual perceptions and the corresponding brightnesses. The validity of the law of coefficients only provides evidence for the supposition that the change in ρ is determined in a primary stage of the visual process, where the response is *still* linear with the stimulus. If for instance ρ were determined by the relative abundance of some photosensitive substance the validity of the law of coefficients would be a trivial matter.

No conclusion can be drawn nor can any supposition be made concerning the relation between the response in further stages of the visual process and the original stimulus.

The validity of the law of coefficients implies that it is immaterial whether a match is made by raising the illumination of the test object or by lowering the illumination of the comparison object. In all experiments the first procedure was followed.

That no discrepancies are found even at the lowest brightness of 0.3 c/m² proves moreover that the veil of light does not yet play an appreciable rôle in the experiments.

A second important law relating to the drop in sensitivity is the *law of constant level*. Apart from a gradual fall, not exceeding 30 percent of ρ, no difference was found when ρ was measured at times ranging between 4 and 600 seconds after the beginning of the exposure. As has been described above, the rate of recovery from this constant level was found to be strongly dependent upon the time of exposure.

We may thus distinguish between two kinds of processes occurring in the retina under influence of the glare source. These processes will be referred to, so as not to anticipate any theoretical interpretation, as *α-adaptation* and *β-adaptation*.

The α-process or α-adaptation which reaches a stationary value within a fraction of a second manifests itself in a sudden drop in sensitivity of the whole of the retina at the beginning of the exposure of a small area and in a very quick rise of sensitivity after short exposures. The subsequent β-process or β-adaptation does not reach its stationary value until after three to five minutes. It manifests itself in the gradual slowing down of the rate of recovery after longer times of exposure but does not manifest itself in the sensitivity during exposure with an incandescent lamp as glare source.

This classification is reminiscent of the classification of after images. These are also strongly different after short flashes of light and after longer times of fixation of an object. In terms of after images the experiments on the recovery of the sensitivity described here should be expressed as measurements of the direct as well as of the indirect negative after image of the glare source.

The Nature of the α-Process

As to the origin of the effect described here, consisting in an adaptive influence undergone by the fovea, we are left with two possibilities of

[13] J. von Kries, "Theoretische Studien über die Umstimmung des Sehorgans," Festschr. Univ. Freiburg 143–158 (1902).

explanation. The first (I) is that of an influence originating in the illuminated parafoveal area and transmitted by the retina towards the fovea. A special form of this possibility—nervous interaction—has been put forward generally to explain experiments such as described above. There is, however, another hypothetical phenomenon which might account for the observed effects. This second possibility (VI) is that of an adaptation caused by the extra veil of light (II, III) falling on the fovea. As the brightness of this veil of light is mostly much lower than that of the test field itself and as moreover the same effect is measured with a very large test field of such brightness that the veil of light is scarcely perceptible, it would seem impossible that this small amount of light might exercise so great an influence. There can, however, be one physical difference between the light from the test object and the light due to scattering; namely, the angle of incidence on the retina. This reminds one immediately of the Stiles-Crawford[14] effect, which consists of a difference in brightness between objects seen by light rays falling perpendicularly or obliquely on the retina, the brightness being considerably lower in the latter case. Apart from other explanations the possibility exists that rays falling obliquely on the retina have another relative action upon the excitatory and the inhibitive processes in favor of the latter. This could result in a lower sensitivity and thus might account for some of the observed phenomena.

It is possible, however, to prove the existence of an influence transmitted by the retina. A few years ago one of us[15] described a phenomenon which strongly resembles the α-adaptation referred to above. This phenomenon was observed when taking measurements of the transparency of the sclerotic as a function of the wave-length of the incident light. If light is thrown, for instance, on the temporal part of the sclerotic the whole field of view with the exception of the shadows of the blood-vessels is seen homogeneously illuminated (V, diascleral re-illumination). The experimental arrangement was such that homogeneous light was thrown alternately on a point of the sclerotic and on a piece of diffusing glass placed immediately in front of the eye. By adjusting a photometric wedge placed in one of the light paths the two brightnesses could be matched in successive contrast. The ratio of intensities necessary for different wave-lengths immediately gives the transparency of the sclerotic as a function of the wave-length. In the course of these experiments a very curious phenomenon was observed. The brightness perception of the diascleral re-illumination fell markedly during the first tenth of a second. The brightness due to the diffusing glass showed no such effect. Thus a different match was obtained for the transition of the glass to the sclerotic and for the transition of the sclerotic to the glass. The relation of the two adjustments was a measure of the adaptive effect. In these experiments the same two possibilities of explanation present themselves. There exists either an influence originating in the exposed peripheral part of the retina transmitted by the retina towards the fovea (IV), or a heightened adaptive influence of the light rays falling obliquely on the retina (V+VI).

A modification of this experiment makes it possible to decide between these two possibilities. The retina ends at the ora serrata corresponding, on the sclerotic, with a line parallel to the outer edge of the iris at a distance of 8 mm. Thus light passing the sclerotic between this outer edge and the ora serrata will not pass the retina and therefore cannot cause the retinal effect IV. Light passing the sclerotic beyond this limit, however, will pass the peripheral retina and thus may cause the retinal effect. Measurements were carried out with violet light ($\lambda = 435.8$ mμ). The effect, as can be judged from Fig. 4 does not exist at distances smaller than 8 mm from the outer edge of the iris, beyond that limit it immediately occurs. This proves that the adaptive effect is not due to a specific adaptive influence of the

[14] W. S. Stiles and B. H. Crawford, "The luminous efficiency of rays entering the eye pupil at different points," Proc. Roy. Soc. **B112**, 428–450 (1933). W. S. Stiles, "The luminous efficiency of monochromatic rays entering the eye pupil at different points and a new colour effect," Proc. Roy. Soc. **B123**, 90–118 (1937). B. H. Crawford, "The luminous efficiency of light entering the eye pupil at different points and its relation to brightness threshold measurements," Proc. Roy. Soc. **B124**, 81–96 (1937). W. D. Wright and J. H. Nelson, "The relation between the apparent intensity of a beam of light and the angle at which the beam strikes the retina," Proc. Phys. Soc. **48**, 401–405 (1936).

[15] J. F. Schouten, "Eine entoptische Methode zur Bestimmung der spektralen Durchlässigkeit der Augapfelwand," Proc. K. Akad. Amsterdam **37**, 516–520 (1934).

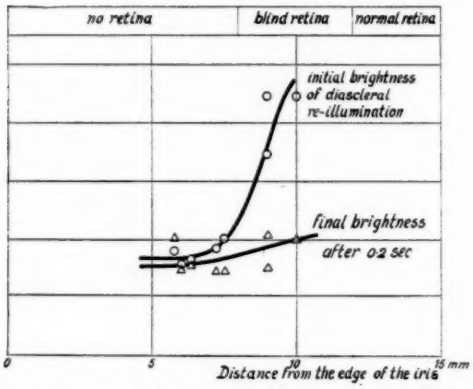

FIG. 4. Curves showing the brightness of diascleral re-illumination as a function of the position of the light spot on the sclerotic. Arbitrary scale of brightness.

oblique light rays but to a retinal influence. A rather remarkable conclusion which may moreover be drawn from these measurements is the following. The effect is shown to begin at the ora serrata. The utmost peripheral part of the retina, ranging from about 8–12 mm can be shown to be absolutely blind, although no degeneration of the retina can be ascertained anatomically. These experiments prove that this blind part of the retina is still in possession of its inhibitive faculties and provide another argument for the hypothesis that these inhibitive functions are situated in one of the earliest stages of the visual process. A control experiment was carried out by alternately illuminating a point between the edge of the iris and the ora serrata and a point beyond the ora serrata. No adjustment of the diascleral re-illumination could be made such that both transitions matched. The effect again existed when illuminating the latter and scarcely, if at all, existed when illuminating the former point.

We are now justified in concluding: The α-process, which manifests itself in a state of lowered sensitivity (α-adaptation), originates in the illuminated part of the retina and is transmitted by the retina, or at least by some medium within the eye, towards the fovea.

This result by no means excludes the possibility of an adaptive explanation of the Stiles-Crawford effect. On the contrary, this possibility should well be kept in mind with all experiments on adaptive effects.

The experiments thus far described deal with the sensitivity of the fovea, the center of the circle in Fig. 3 being rigidly fixated during experiments. Experiments were also carried out with other points of fixation, giving the state of adaptation of other, parafoveal, parts of the retina. In Fig. 5 the values of the glare index V are reproduced at the corresponding point of fixation. First, it is seen that the glare index is lower when the glare source is fixated instead of the test object. This means that the fovea is influenced more by its surroundings than the surroundings by the fovea.

This rule not only holds true when the glare source is placed to the right of the objects, but also when it is placed to the left, above or below these. Therefore the effect is not due to an asymmetrical behavior of the retina but to the relative position of the image of the glare source and the test object on the retina.

Second, it is seen from Fig. 5 that the glare index rises when the point of fixation travels upwards or downwards. This means that regions outside the fovea have a greater influence upon each other than regions in or near the fovea. These experiments moreover provide a second disproof of the explanation of α-adaptation as due to the obliquely entering scattered rays. The angle of incidence upon the image of the test object is independent of the point of fixation. Whether now the eye fixates the point L or the points marked with 5.6 the distance of the image of the test object from the fovea is the same. As

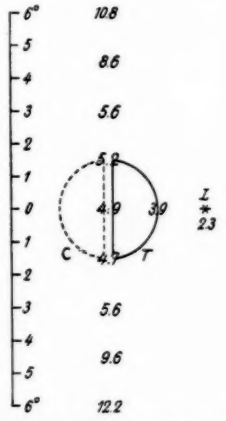

FIG. 5. Values of the glare index V for different points of fixation. Each value of V is represented at the corresponding point of fixation.

the retina does not behave in markedly asymmetrical manner, the glare indices in that case ought to be expected to be the same. The experiment clearly shows the opposite result. Fixation of L in fact gives a glare index below, fixation of the points 5.6 a glare index above the foveal value.

It should be expected, according to our hypothesis of the α-adaptation that anomalies would occur when the blind spot is illuminated. No such anomalies, however, could be ascertained with certainty. The total blindness of the blind spot being still a matter of dispute, this result need not be taken as conclusive evidence against the existence of a retinal influence.

Quantitative Measurements of the Sensitivity Level During Exposure

The fall in sensitivity at the beginning of the exposure takes place too quickly to be measured with the normal method. Therefore use was made of a slit-wheel (Figs. 2 and 6). This wheel contains two slits of variable width and relative position and is placed in such a way that the glare source L becomes visible through slit S_1, the two objects through slit S_2. The time of revolution used was 1.4 sec. The time of illumination by the glare source L was varied between 20 and 200σ (0.02 − 0.2 sec.) by varying the width of S_1. By means of slit S_2 the two objects were made visible during 10σ at the end of the exposure. It is possible in this way to measure the sensitivity of the fovea at the end of very short exposures. Care was taken to avoid summative effects. As can be seen from Fig. 7 the sensitivity drops to a lower value within one tenth of a second. This confirms our qualitative expectations from the diascleral measurements. The level arrived at after 100σ is practically the same as, mostly a little lower than, the level measured by the continuous method after some four seconds of exposure. It is rather surprising that the constant level is reached without overshooting. The brightness of a photometric field as a function of the time was in fact proved by Broca and Sulzer[16] to show appreciable overshooting, the quasi-stationary value being reached after passing a maximum several times higher than this value. Their results were confirmed by similar measurements with the slit-wheel and are moreover in complete accordance with measurements on action currents.

To account for this different behavior we must consider that the visual process in the retina takes place by different stages. In the first stage radiant energy is absorbed. The energy is transformed and passed on to a second stage and so forth until in the last retinal stage the nerve fiber is stimulated. The different and simpler dependence of α-adaptation upon the time as compared to brightness perception seems to indicate that the brightness is by no means the quality determining the influence induced in the surrounding retina but that α-adaptation is either determined by one of the preceding stages or by another separate retinal process.

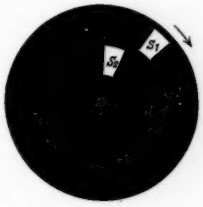

FIG. 6. The slit-wheel. The glare source L becomes visible through slit S_1, whereas the test object T and the object of comparison C become visible through slit S_2.

An attempt was made to measure the velocity of transmission of α-adaptation over the retina. Slits S_1 and S_2 were both set at 10σ. Thus a very short flash from the glare source was obtained. By varying the distance of the slits the sensitivity of the fovea could be determined as a function of the time elapsed after the flash. Measurements were taken with a glare angle of 10° and with a very strong flash on the sclerotic. Any effect of finite time of transmission would manifest itself in a time lag between the two curves thus obtained. For both curves the minimum sensitivity occurred 50σ after the light flash, whereas after 150σ the sensitivity was again at the original level. No difference between the two curves could be established. Therefore the time of transmission is not larger than the relative error made in obtaining the two curves. This error is about 20σ.

The level of sensitivity ρ was measured with

[16] A. Broca and D. Sulzer, "La sensation lumineuse en fonction du temps," J. physiol. et pathol. gén. **4**, 632–640 (1902).

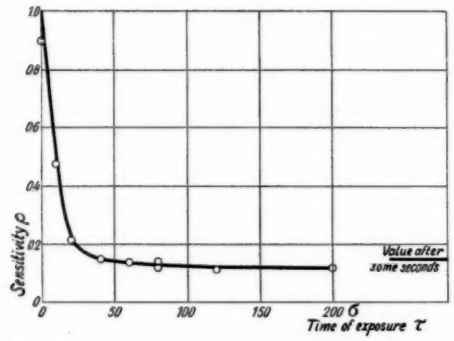

FIG. 7. Curve showing dependence of the sensitivity upon the time as measured with the slit-wheel, during the first 0.2 sec. of exposure. Glare angle $\varphi=6°$. Illumination at the eye 180 lux. Each point gives the mean of four readings.

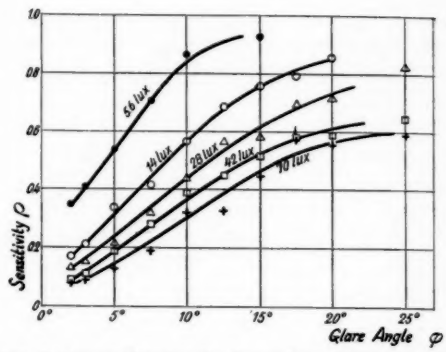

FIG. 8. Curves showing the dependence of the sensitivity ρ on the glare angle φ for different values of the illumination at the eye E.

the continuous method as a function of the illumination E upon the eye and the glare angle φ. Because of the law of constant level the time at which the measurements are taken is practically immaterial. All readings were taken five seconds after the beginning of the exposure. Fig. 8 represents a series of such measurements: each point gives the mean value of four readings. It is seen that the drop in sensitivity decreases rapidly with increasing glare angle φ, and increases with higher illumination E. The former result would suggest that the illumination of an element of the retina produces a sort of pit in the sensitivity of the surrounding retina. The same experimental result, however, would be obtained if illumination of a retinal element produced a uniform drop in sensitivity over the whole retina but to a less degree with varying distance of the illuminated point from the fovea.

The existence of the pit, however, can be proved by fixating the glare source instead of the test object and taking measurements at varying values of φ. In this case the illuminated part of the retina is fixed and the sensitivity at varying distances from this point is measured. The same dependence of ρ upon the glare angle φ is found. This proves the existence of the "pit."

It is found that the glare index V bears a simple relation to the illumination E on the eye and the glare angle φ; namely,

$$V=P(E/\varphi). \tag{3}$$

P is a constant which is equal to about 0.4 if E is expressed in lux and φ in degrees. In Table I values of P are given for various values of E and two values of φ. It is seen that P is generally a little smaller for higher values of E, which can partly be accounted for by correcting for the varying area of the pupil. P is generally a little smaller for the higher values of φ. As the difference in area of the pupil would have the opposite result we may conclude that the power of φ in (3) ought to be taken a little higher than 1.

The linear relation between V and E, which was also found to hold true for direct adaptation by extrapolating curves of the recovery of the sensitivity, is in fact a generalization of Wright's relation. Wright found that for direct adaptation the sensitivity level is inversely proportional to the brightness of the glare source. This was found to hold true with good precision for sensitivities below $\rho=0.2$. This rule evidently cannot hold true for higher values of ρ because for $E=0$ the sensitivity ρ must be equal to 1; whereas, Wright's relation would lead to ∞. Formula (3) accounts for this discrepancy. For low values of ρ, as can be seen from (2), the glare index becomes practically inversely proportional to ρ. In that region Wright's relation is a good approximation.

Quantitative Measurements of the Sensitivity After Exposure

We paid considerable attention to measurements of the recovery of the sensitivity after direct, indirect and diascleral adaptation. Wright's law, implying that for direct adaptation the recovery and thus also the extrapolated value ρ_s at

the end of the exposure is independent of the size of the adapting light source, was found to hold true for diameters ranging between 4 and 24 degrees. His suggestion, that whatever retinal mechanism is responsible for the process of adaptation, its action must be confined to the particular area stimulated, must be considered wrong in this general form. But as it has been proved that retinal interaction exists, the validity of his law only gains in importance. The recovery curves of ρ against the time for different intensities, and for a time of exposure τ of 60 seconds for indirect as well as for direct adaptation, run practically parallel (Fig. 9). This confirms Wright's results with direct adaptation. For different times of exposure, however, such as is shown in Fig. 10, the curves show a considerable dependence of the slope upon the time of exposure. This especially follows from measurements on indirect and diascleral adaptation, where the constant value of ρ during exposure (ρ_s) could be measured. We thus see that different curves of recovery can exist for the same sensitivity level ρ_s. The most general conclusion to be drawn from this statement is that the recovery and thus the state of adaptation of the retina is determined by at least two independent parameters.

For direct adaptation, the value of ρ_s can only be obtained by extrapolation, a procedure which is very uncertain for short times of exposure. There exists, however, a striking resemblance between the recovery curves after direct and those after indirect or diascleral adaptation. The recovery of direct adaptation has been mostly interpreted hitherto as being of a photochemical nature. Indirect adaptation has now been demonstrated which is not initiated by light falling on that part of the retina. Therefore the recovery can scarcely be interpreted as being of photochemical nature and can probably better be interpreted as being of electrochemical or electrokinetic nature. The close resemblance between the curves then suggests that direct adaptation is also of the latter and not of the former type.

No theory seems to present itself at the moment which could be tested quantitatively by these recovery curves. However dangerous the construction of empirical relations may be, it should be mentioned that the curves are remarkably well represented by

$$1-\rho=(1-\rho_s)(\tau/(\tau+t)), \qquad (4)$$

where τ stands for the time of exposure and t for the time elapsed after the exposure. The agreement may be judged from Fig. 11, where $\log(1-\rho)/\tau$ obtained from the direct recovery curves in Fig. 10 is plotted against $\log(\tau+t)$. The straight line gives the theoretical value for $\rho_s=0.09$. The relation (4) holds true for values of $\tau+t$ up to 250 seconds and especially for values of ρ_s smaller than 0.10. Relation (4) is the more striking because differentiating leads to

$$\frac{d(1-\rho)}{dt}=-\frac{(1-\rho)^2}{(1-\rho_s)\tau}, \qquad (5)$$

which is a differential equation of the same type as was obtained in 1920 by Hecht[17] for dark

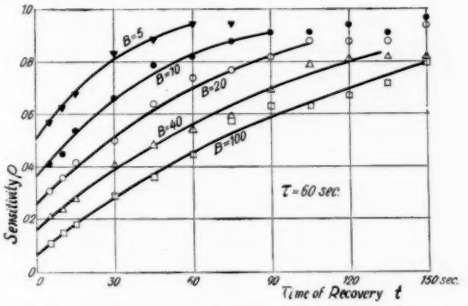

FIG. 9. Curves showing the recovery of the sensitivity ρ after direct adaptation as a function of the time of recovery t, for different brightnesses of the glare source. $B=100$ corresponds with 900 candles/m² (5600 photons). Time of exposure $\tau=60$ sec.

TABLE I. *Values of P in Eq. (3). The experiments are taken over a period of 5 months.*

Glare Angle	3 degrees					5 degrees					See Fig.
Illumination at the Right Eye in Lux	25	50	75	100	200	25	50	75	100	200	
Experiment 66	0.67	0.53	0.41			0.60	0.51	0.42			8
71	0.61	0.57				0.56	0.52				
92		0.49					0.49				
92	0.46	0.43									
97	0.24	0.22	0.24	0.23							
204	0.48	0.43	0.41	0.39	0.35	0.32	0.32	0.29	0.30	0.28	
196				0.25							10

[17] S. Hecht, "The dark adaptation of the human eye," J. Gen. Physiol. **2**, 499–517 (1920). S. Hecht, "The nature of foveal dark adaptation," J. Gen. Physiol. **4**, 113–139 (1922).

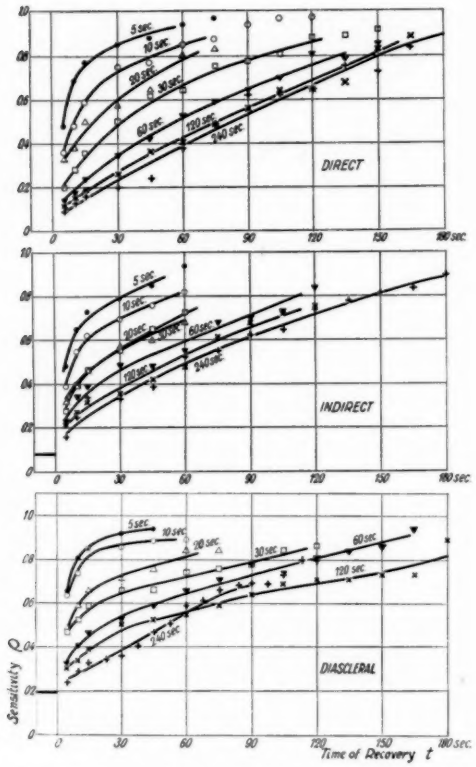

FIG. 10. Curves showing the recovery of the sensitivity ρ as a function of the time of recovery t, for different times of exposure τ. *Upper set of curves:* Direct adaptation. Brightness 710 candles/m² (4600 photons). *Middle set of curves:* Indirect adaptation. Glare angle 3°. Illumination at the eye 130 lux. *Lower set of curves:* Diascleral adaptation. Illumination at the eye 2900 lux.

adaptation and which was interpreted as a bimolecular reaction.*

Equation (4) implies that ρ_s during exposure is independent of the time also for direct adaptation. This evidently cannot be true. The brightness of a light source in fact drops gradually to about $\frac{1}{3}$ of its original value during the first minute of fixation. This can easily be measured by binocular comparison. This means a gradual drop in sensitivity such as was reported moreover by Wright. The seemingly constant level found in these experiments for direct adaptation is due to the following experimental circumstance. For obtaining direct adaptation lamp F is switched on. This lamp is seen reflected in the piece of glass G. During exposure the right eye does not perceive the test field. The left eye, however, views the comparison field and is thus adapted to it during the same time as is the right eye to the glare source. As, notwithstanding the great difference in brightness, this local adaptation is of the same order, the effect is thereby eliminated from the measurements. The values of ρ for direct adaptation in Fig. 10 thus do not give the actual sensitivity, but the sensitivity corrected for the local adaptation. The local adaptation of the test field was proved for incandescent-lamp light to be independent of the state of extraneous adaptation due to the indirect glare source. We thus find a characterizing difference between the behavior of the illuminated and the nonilluminated part of the retina. In the illuminated area the sensitivity quickly drops during the first second of exposure and then gradually falls to a lower level during the first minutes. In the neighborhood the sensitivity immediately drops to a constant level without any appreciable subsequent fall or rise.

The α-process is thus transmitted towards the neighborhood, whereas the processes responsible for the local adaptation seem to be restricted (at least for incandescent-lamp light) to the stimulated area. This, combined with the fact that for incandescent-lamp light the local adaptation to the photometric fields is widely independent of the state of extraneous adaptation, and with the lack of dependence of local adaptation on brightness of the field seems to indicate that the process involved is of a different nature than the α-process. Probably it is seated in a later stage in the chain of visual processes.

THE MECHANISM OF THE α-PROCESS

We have seen that α-adaptation is a reversible state of diminished sensitivity which can be established within 0.1 sec. over practically the whole retina. Most experiments on direct adaptation can be accounted for qualitatively by assuming photochemical processes. To account for indirect adaptation we should have to assume the existence of lateral diffusion of photochemical

* At first this agreement seemed to us not to be perfect because Hecht's relation is stated for a quantity which is in our terminology equal to $(1-\rho)/\rho$ instead of $(1-\rho)$. It can easily be proved, however, that as regards the form of the differential equation this amounts to exactly the same thing.

products from, or of fresh photosensitive substances towards, the illuminated area. The rapidity with which the effect is now shown to take place seems to be a strong argument against any such hypothesis. It seems much more hopeful to connect the results obtained here with our knowledge of the electrophysiology of the eye.

When the eye is illuminated, action currents can be measured which have their origin in the retina. Conversely we know that electrical stimulation of the eye can produce a blue-violet light perception. If the current is reversed, no light perception occurs, but a white object appears darker than before. This indicates that the sensitivity is lowered.

It is thus a fact that illumination of the eye produces electric currents in the eye, and conversely that electric currents in the eye *can* bring about a diminished sensitivity. It therefore seems justifiable to combine these two facts and to propose the following hypothesis: ***When the retina is illuminated there is induced at the illuminated part as also in the surrounding area, an electric state which manifests itself as to the light-sense by a diminished sensitivity.***

This hypothesis immediately accounts for the great rapidity of the α-process and moreover leaves wide scope for further connections between adaptive and electrical phenomena in the eye. This specially applies to the analysis of the action currents. The action current namely can be analyzed into three components with different dependence upon the time. One of these, P_{II}, as has been made specially clear by the work of Granit[18] and others, is identical with brightness perception (Fig. 12). α-adaptation bears a remarkable resemblance to Granit's component P_{III}, first, as regards its dependence upon the time (Fig. 7), second, because of its inhibitory character, and third, because P_{III} seems to be due to a process restricted to the retina.

This explains why Fröhlich[19] measured action currents from points of the retina beside the illuminated area. The experiments of Kohlrausch and Sachs[20] show that for the eye of the dove and for the human eye the action current is predominately negative for light of short wavelength. This would indicate a relatively strong presence of P_{III}. Thus α-adaptation ought to be relatively more pronounced for short wavelengths. Although the observed strong adaptive effect for blue diascleral adaptation points in this direction, no marked effect has otherwise as yet been established.

If really the diminished sensitivity be caused by an electrical phenomenon of a certain sign, we might expect that under special circumstances electrical phenomena of the other sign would occur in the eye. These would manifest themselves in a light perception of probably blue-violet color, according to our knowledge of electrical stimulation of the eye. In respect with this it is curious to consider that the perception of blue plays an exceptional rôle in practically every aspect of vision: the blue arcs, the bluish color of the secondary and the tertiary after image, the relative abundance of blue perception when the eye is strongly pressed, the blue effect

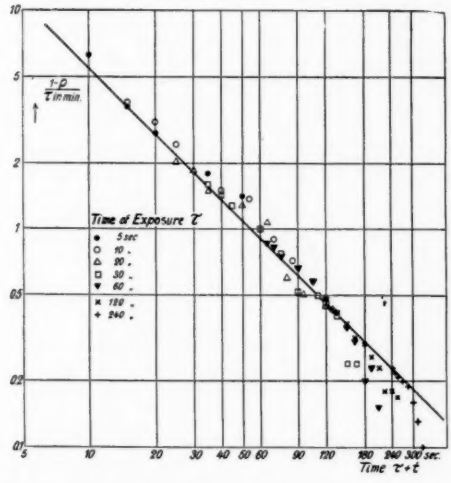

FIG. 11. Verification of the empirical relation 4, with the values taken from Fig. 10 (upper set of curves). Log $(1-\rho)/\tau$ is plotted against log $(\tau+t)$. The straight line is the theoretical line for $\rho_s=0.09$.

[18] See e.g. R. Granit, "Die Elektrophysiologie der Netzhaut und des Sehnerven," Acta Ophth. Suppl. VIII, 14, 1–98 (1936).

[19] F. W. Fröhlich, "Beiträge zur allgemeinen Physiologie der Sinnesorgane," Zeits. Sinnesphysiol. **48**, 28–164, 354–438 (1914).

[20] A. Kohlrausch, "Elektrische Erscheinungen am Auge," Bethes Handb. Norm. Path. Physiol. **12**, No. 2, II, 1394–1496 (1931).

with sodium light* (Bouma[21]), the blue halo surrounding a neon light source on dark background, the exceptional rôle of the blue sensation curve, etc.

Newhall[22] brought new evidence in favor of the electrical nature of the blue arcs, whereas the positive after images have often been interpreted as being of electrical origin. The blue effect with sodium light and the blue halo with neon light can be caused to disappear immediately and completely by suitable electrical stimulation of the eye. On the other hand we have not succeeded in the reverse sense in suppressing the blue-light perception caused by electrical stimulation by indirect blinding.

No remaining depressed sensitivity could be ascertained after the eye had been electrically stimulated so as to produce a diminished sensitivity. On the contrary a strong off-effect consisting in a blue perception of short duration always occurred. In our terminology this means that the "α-adaptation" which is brought about by a general electric field in the eye is not accompanied by "β-adaptation."

Therefore the general electric field which is brought about in the eye by illuminating a certain area of the retina and which manifests itself in the action current can but partly account for the observed adaptive effects.

We are thus, after excluding all possibilities of purely physical or chemical nature, led to assume that indeed the retinal synapses are, at least partly, the seat of the adaptive influence of an illuminated area of the retina on its surroundings. This assumption moreover is necessary to account for the effects observed with chromatic light.

Experiments with Strongly Chromatic Light

The effects with glare source and objects of special spectral composition have only been qualitatively investigated. A few important conclusions can none the less be drawn. When the eye is illuminated by red light, using white objects, the test field immediately drops in brightness but scarcely changes in chromaticity. Gradually, however, the chromaticity changes until after a few minutes the test field has taken on a saturated green color. After illumination the chromaticity and the general sensitivity gradually return to their original value. The gradual change in chromaticity corresponding to a relatively large drop in sensitivity for red light was expected from well-known phenomena of fatigue.

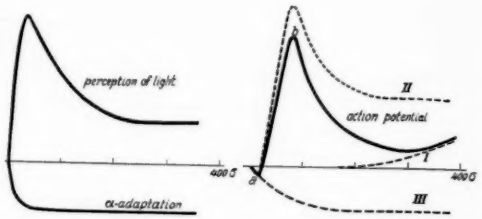

FIG. 12. Resemblance between the adaptive and the electrical properties of the eye. *Left.* Upper curve. The dependence of the brightness perception of an object of 5 candles/m² upon the time. Measured with the slit-wheel. Lower curve. The dependence of the sensitivity upon the time (see Fig. 7). *Right.* Analysis of the action potential of the dark-adapted eye of a cat. (After Granit.)

From the fact that the initial drop in sensitivity is practically the same for all wave-lengths we at first drew the seemingly obvious conclusion that α-adaptation is a state of generally depressed sensitivity, and thus a function of one parameter only. That would accord with the hypothesis that α-adaptation is caused by the electric field in the eye.

Further experiments, however, show that the phenomena are much more complicated. If for example the test field is only shown momentarily it is seen in a saturated green color. This proves, in any case, that the α-adaptation with red glare source consists in a relatively large drop of the sensitivity for red light. A contradiction seems to exist between this experiment and the one first described. This may be resolved as follows. If the white object is fixated during a few seconds and the point of fixation is shifted, an after image of the test object is seen on the red straylight surrounding the glare source. This after image which lasts for a few seconds is darker and more saturated than the red straylight. This proves the existence of a local adaptation due to the test

* An outstanding peculiarity of this effect, which cannot be accounted for by any customary process of "fatigue," is that a small admixture of white light causes the effect to disappear immediately and completely.

[21] P. J. Bouma, "Phenomena of eye fatigue and after images with monochromatic light," Physica **1**, 429–436 (1934).

[22] S. M. Newhall, "The Constancy of the Blue Arc Phenomenon," J. Opt. Soc. Am. **27**, 165–176 (1937).

field, consisting in a diminished sensitivity for the spectral region complementary to that of the glare source.

This leads to the following interpretation: The red glare source causes a drop in sensitivity for red in the surrounding retina. If, however, the retina in that state of extraneous adaptation views a white object illuminated by incandescent-lamp light, a secondary local adaptation occurs in such measure that the ultimate sensitivity is, at least for the present, no more dependent upon the wave-length.

In another experiment a blue glare source was used, obtained by a filter which cut off all wave-lengths above 550 mμ. Two pairs of pigmented papers, a red and a blue, served as objects, and enabled us to compare the sensitivity for the red and the blue region simultaneously. If the objects are viewed momentarily the blue field is seen much darker than the red field whereas the red field remains of practically the same brightness.

If, however, the fields are viewed continuously a match can be made which is practically the same for the two pairs, indicating that the sensitivity is equally depressed for both. If the glare source is turned off after one minute's exposure the blue field gradually returns to its original brightness, in very much the same way as was described for incandescent-lamp light. The red field, however, notwithstanding the long time of exposure, shows a striking jump in brightness and reaches the original value in a much shorter time. Here we have again the drop in sensitivity for the spectral region of the glare source, caused by the extraneous α-process, and the subsequent drop in sensitivity for the remaining spectral region caused by the secondary local process. Together they again result in a uniformly depressed sensitivity level.

These two experiments give a good illustration of the fact that the ultimate sensitivity of a point of the fovea is determined by extraneous adaptation caused by the glare source as well as by local adaptation caused by the test object itself.

For strongly chromatic light this local adaptation is shown to be linked up in a queer way with extraneous adaptation. For the weakly chromatic incandescent-lamp light, however, local and extraneous adaptation seem to be widely independent of each other. From the physical point of view it seems more adequate to suppose that for weakly chromatic light the interaction effects just counterbalance and thus do not manifest themselves in the experiments.

Mechanism of the β-Process

Once α-adaptation is proved to be caused by nervous interaction, the nature of β-adaptation becomes very difficult to understand. β-adaptation, which manifests itself in the rate of recovery of the sensitivity after exposure and probably also in the gradual change in sensitivity such as was described for a red glare source, seems to be strongly linked with α-adaptation. The primary action of light seems to be a photovoltaic process. Alas, the physics of this process is not yet sufficiently unraveled to be of much help in predicting further consequences. The general behavior of such processes, however, such as can be seen from the experiments of Goldmann[23] and of Vanselow and Sheppard,[24] show remarkable analogies with the effects of fatigue in the eye. In any event theories based alone on photochemical changes became more and more improbable. A migration of photosensitive substances and their products over very small distances under influence of the state of α-adaptation of the retina seems a more likely assumption to explain the secondary adaptive effects.

Rules

The present investigations revealed or confirmed a number of rules governing the visiodynamical phenomena which are summarized below. We wish to postulate neither their ultimate importance, nor their universal and rigorous validity. They may serve solely as a basis for further experiments.

1. Incandescent-lamp light

a. The sensitivity is independent of the brightness of the object of comparison. (*Law of coefficients*) (0.3–5.0 c/m²).

b. The sensitivity is independent of the size of the object ($0.7°\times0.7°-3.6°\times7.2°$).

[23] A. Goldmann, "Lichtelektrische Untersuchungen an Farbstoffzellen," Ann. d. Physik **4**, 27, 449–536 (1908).

[24] W. Vanselow and S. E. Sheppard, "Photo-Voltaic cells with silver-silver bromide electrodes I," J. Phys. Chem. **33**, 331–353 (1929).

c. The level of sensitivity during indirect adaptation is constant (*Law of constant level*) (0.06–600 seconds).

d. For direct adaptation the glare index V is linearly proportional to the brightness of the glare source (generalization of Wright's relation) (0–900 c/m²).

e. For direct adaptation the sensitivity level is independent of the size of the glare source. (Wright's law) (4°–24°).

f. For indirect adaptation the glare index V is directly proportional to the illumination E at the eye (0–200 lux).

g. For indirect adaptation the glare index V is inversely proportional to the glare angle φ (2°–25°).

h. Local adaptation due to the test object and extraneous adaptation due to the glare source show no interaction.

i. The recovery curves for direct adaptation, corrected for local adaptation, are identical with those for indirect adaptation ($\tau = 5 - 240$ sec.).

j. The parafovea is less affected by the fovea than inversely. Two points in the fovea affect each other less than two points outside the fovea.

k. The gradual drop in brightness perception of a photometric field is independent of the physical brightness (5–700 c/m²).

2. Strongly chromatic light

a. α-adaptation is dependent upon the chromaticity of the glare source. It is generally strongest for this chromaticity.

b. Local adaptation due to the test field is dependent upon the state of extraneous adaptation and generally results in a uniformly depressed level of sensitivity.

Summary

The binocular method, generalized for the measurement of indirect adaptation, makes it possible to obtain quantitative data on the adaptive processes in the retina. When a small area of the retina is illuminated, the sensitivity of this area and also of the surroundings drops to a considerably lower level within 0.1 second (α-adaptation). This sudden fall is followed by a gradual drop in brightness of the light source indicating a gradual drop in sensitivity. In the surroundings, however, the sensitivity remains practically constant during exposure. Notwithstanding this fact some processes must be going on in that part of the retina because the rate of recovery of the sensitivity after exposure is dependent upon the time of exposure (β-adaptation). Hence the state of adaptation of the retina is determined by at least two independent parameters.

The observed effects and especially the simple form in which most of the quantitative relations can be expressed lead to the supposition that the adaptive effects are not only of retinal origin but are moreover seated in one of the earliest stages of the chain of retinal processes.

Indirect adaptation can neither be accounted for by assuming a lateral diffusion of photosensitive substances of photolytic products over the retina, nor by assuming a strong adaptive influence of the stray light within the eye.

The hypothesis is put forward that α-adaptation is of electrical origin. This hypothesis has the general advantage of bridging the existing gulf between the adaptive and the photoelectrical properties of the eye.

The adaptive effects, however, can but partly be explained from the general electric field which is known to be brought about in the eye by illuminating a small area of the retina. Therefore, indirect adaptation must have its seat in the retinal synapses. Experiments with strongly chromatic light suggest that these lateral cells are equipped with "color sense."

β-adaptation could hitherto be fairly well described by photochemical theories. The strong relation of β-adaptation with α-adaptation, however, points against such theories and suggests that β-adaptation is also due to the electrical processes in the retina.

The complicated phenomena, occurring when chromatic light is used, illustrate the essentially dynamical character of the processes of vision.

The binocular method promises to be of great use in a further quantitative analysis of the complicated adaptive processes and may lead to a deeper insight into the nature of these processes.

We are indebted to Mr. T. G. Hagedoorn for carrying out control experiments and to Dr. Deane B. Judd for valuable suggestions regarding the paper.